Anomaly of $Zb\bar{b}$ coupling revisited in MSSM and NMSSM

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ABSTRACT: The $Zb\bar{b}$ coupling determined from the Z-pole measurements at LEP/SLD shows an about 3σ deviation from the SM prediction, which would signal the presence of new physics in association with the $Zb\bar{b}$ coupling. In this work we give a comprehensive study for the full one-loop supersymmetric effects on the $Zb\bar{b}$ coupling in both the MSSM and the NMSSM by considering all current constraints which are from the precision electroweak measurements, the direct search for sparticles and Higgs bosons, the stability of Higgs potential, the dark matter relic density, and the muon $g-2$ measurement. We analyze the characters of each type of the corrections and search for the SUSY parameter regions where the corrections could be sizable. We find that the sizable corrections may come from the Higgs sector with light m_A and large $\tan\beta$, which can reach -2% and -6% for ρ_b and $\sin^2\theta_{\text{eff}}^b$, respectively. However, such sizable negative corrections are just opposite to what needed to solve the anomaly. We also scan over the allowed parameter space and investigate to what extent supersymmetry can narrow the discrepancy. We find that under all current constraints, the supersymmetric effects are quite restrained and cannot significantly ameliorate the anomaly of $Zb\bar{b}$ coupling. Compared with $\chi^2/dof = 9.62/2$ in the SM, the MSSM and NMSSM can only improve it to $\chi^2/dof = 8.77/2$ in the allowed parameter space. Therefore, if the anomaly of $Zb\bar{b}$ coupling is not a statistical or systematic problem, it would suggest new physics beyond the MSSM or NMSSM.

KEYWORDS: Supersymmetry Phenomenology.

Contents

1. introduction

Although most of the electroweak data are consistent with the Standard Model (SM) to a remarkable precision, there are still some experimental results difficult to accommodate in the SM framework. A well known example is that the effective electroweak mixing angle $\sin^2 \theta_{\text{eff}}$ determined from the leptonic asymmetry measurements is much lower than the value determined from the hadronic asymmetry measurements [1, 2], and the averaged value over all these asymmetries has a χ^2/dof of 11.8/5, corresponding to a probability of only 3.7% for the asymmetry data to be consistent with the SM hypothesis. Such a large discrepancy mainly stems from the two most precise determinations of $\sin^2 \theta_{\text{eff}}$, namely the measurement of A_{LR} by SLD and the measurement of the bottom forward-backward asymmetry A_{FB}^b at LEP, which give values on opposite sides of the average and differ by 3.2 standard deviation. It is interesting to note that if such a discrepancy is attributed to experimental origin and thus the hadronic asymmetry measurements are not included in the global fit, then a rather light Higgs boson around 50 GeV is indicated from the fit [3, 4], which is in sharp contrast with the LEP II direct search limit of 114 GeV [5]

and results in a compatible probability as low as 3%. If we resort to new physics to solve this discrepancy, the new physics effects must significantly modify the $Zb\bar{b}$ coupling while maintain the Z-boson couplings to other fermions basically unchanged. In this work we focus on the $Zb\bar{b}$ coupling and scrutinize the supersymmetric effects.

In our analysis we choose to parameterize the $Zf\bar{f}$ interaction at Z-pole in term of the parameter ρ_f and effective electroweak mixing angle $\sin^2\theta_{\rm eff}^f$ [6, 7]:

$$\Gamma_{Zf\bar{f}}^{\mu} = (\sqrt{2}G_{\mu}\rho_f)^{\frac{1}{2}} m_Z \gamma^{\mu} \left[-2Q_f \sin^2\theta_{\rm eff}^f + I_3^f(1 - \gamma_5) \right] \qquad (1.1)$$

This parametrization is preferred from the experimental point of view because all the measured asymmetries are only dependent on $\sin^2\theta_{\rm eff}^f$ and their precise measurements can directly determine the value of $\sin^2\theta_{\rm eff}^f$. From the combined LEP and SLD data analysis, the fitted values of ρ_f and $\sin^2\theta_{\rm eff}^f$ agree well with their SM predictions for leptons and light quarks, but for the bottom quark their fitted values are respectively 1.059 ± 0.021 and 0.281 ± 0.016 (with correlation coefficient 0.99), which significantly deviate from their SM predictions of 0.994 and 0.233 (for $m_t = 174\,{\rm GeV}$ and $m_h = 115\,{\rm GeV}$) and leads to $\chi^2/dof = 9.62/2$ (corresponding to a compatible probability of 0.8%). To best fit the experimental data, ρ_b and $\sin^2\theta_{\rm eff}^b$ should be enhanced by about 6.5% and 20%, respectively. While we can envisage that the supersymmetric effects are not usually so large, we want to figure out to what extent supersymmetry can improve the situation. For this purpose, we choose two popular supersymmetric models: the minimal supersymmetric model (MSSM) [8] and the next-to-minimal supersymmetric model (NMSSM) [9].

For the NMSSM effects on $Zb\bar{b}$ coupling, which have not been studied in the literature, we will perform the calculation to one-loop level. For the MSSM effects, which have been studied by many authors [10–12], we will renew the study in the parametrization of ρ_b and $\sin^2\theta_{\rm eff}^b$ (the previous studies usually examined the effects on the Z-width, the ratio R_b and the asymmetry A_{FB}^b). For both the MSSM and NMSSM, we will consider various current experimental constraints on the parameter space, which are from the precision electroweak measurements, the direct search for sparticles and Higgs bosons, the stability of the Higgs potential, the cosmic dark matter relic density, and the muon g-2 measurement.

This paper is organized as follows. In sectionII we introduce the general formula for the calculation of ρ_f and $\sin^2\theta_{\rm eff}^f$ and apply them to the MSSM and NMSSM. In sectionIII we summarize the constraints considered in this work and briefly discuss their characters. In section IV and section V we perform numerical study for the corrections to ρ_b and $\sin^2\theta_{\rm eff}^b$ in the MSSM and NMSSM, respectively. We will first show the characters of different type corrections, then we will scan the whole SUSY parameter space to investigate the compatibility of the supersymmetric predictions of ρ_b and $\sin^2\theta_{\rm eff}^b$ with their experimental results. Finally, in section VI we conclude our work with an outlook on the possibility of solving the $Zb\bar{b}$ anomaly.

2. General formula to calculate ρ_f and $\sin^2\theta_{\rm eff}^f$

In the SM with the input parameters the Fermi constant G_F, the fine-structure constant α, Z-boson mass m_Z and fermion masses m_f, the electroweak mixing angle $s_W = \sin\theta_W$

is determined at loop level by [13–15]

$$s_W^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu m_Z^2}\frac{1}{1-\Delta r}}\right) \tag{2.1}$$

where Δr is given by

$$\Delta r = \frac{\hat{\Sigma}^W(0)}{m_W^2} + \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7 - 4s_W^2}{2s_W^2}\ln(1-s_W^2)\right) + 2\delta^v + \delta^b \tag{2.2}$$

with $\hat{\Sigma}^W$ denoting the renormalized W-boson self-energy, δ^v and δ^b being the vertex correction and box diagram correction to μ decay $\mu \to \nu_\mu e\bar{\nu}_e$, respectively. To get a more precise numerical result for s_W^2, one can iterate eqs. (2.1) and (2.2) a few times.

With the s_W defined above, the effective $Zf\bar{f}$ coupling at Z-pole takes the following form [7, 14]

$$\Gamma^\mu_{Zf\bar{f}} = \left(\sqrt{2}G_\mu(1-\Delta r)\right)^{\frac{1}{2}} m_Z\gamma^\mu\Big\{v_f - a_f\gamma_5 + \delta v_f - \delta a_f\gamma_5$$
$$-\frac{1}{2}\left[\Sigma'_Z(m_Z^2) + \delta Z_2^Z\right](v_f - a_f\gamma_5) - 2Q_f s_W^2\Delta\kappa\Big\}, \tag{2.3}$$

where $v_f = I_3^f - 2Q_f s_W^2$ and $a_f = I_3^f$ are respectively the vector and axial vector coupling coefficients of $Zf\bar{f}$ interaction at tree level, and δv_f and δa_f are their corresponding corrections. Σ'_Z is the derivative of the unrenormalized Z-boson self-energy Σ_Z with respect to the squared momentum p^2, and δZ_2^Z is the field renormalization constant of Z-boson given by

$$\delta Z_2^Z = -\Sigma'_\gamma(0) - 2\frac{c_W^2 - s_W^2}{s_W c_W}\frac{\Sigma_{\gamma Z}(0)}{m_Z^2} + \frac{c_W^2 - s_W^2}{s_W^2}\left(\frac{Re\Sigma_Z(m_Z^2)}{m_Z^2} - \frac{Re\Sigma_W(m_W^2)}{m_W^2}\right), \tag{2.4}$$

and $\Delta\kappa$ is given by

$$\Delta\kappa = \frac{c_W^2}{s_W^2}\left\{\frac{\Sigma_Z(m_Z^2)}{m_Z^2} - \frac{\Sigma_W(m_W^2)}{m_W^2} - \frac{s_W}{c_W}\frac{\Sigma_{\gamma Z}(m_Z^2) + \Sigma_{\gamma Z}(0)}{m_Z^2}\right\}. \tag{2.5}$$

In eq. (2.3) the factor $\frac{1}{2}(\Sigma'_Z(m_Z^2) + \delta Z_2^Z)$ comes from the fact that the residue of the renormalized Z propagator is different from 1, while the last term enters due to $Z-\gamma$ mixing at Z-pole.

If we re-express $\Gamma^\mu_{Zf\bar{f}}$ in eq. (2.3) in term of ρ_f and $\sin\theta^f_{\text{eff}}$ as in eq. (1.1), we get

$$\rho_f = 1 + \delta\rho_{se} + \delta\rho_{f,v}, \tag{2.6}$$
$$\sin^2\theta^f_{\text{eff}} = (1 + \delta\kappa_{se} + \delta\kappa_{f,v})s_W^2, \tag{2.7}$$

with $\delta\kappa_{se} = \Delta\kappa$ and

$$\delta\rho_{se} = \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2} - 2\frac{s_W}{c_W}\frac{\Sigma_{\gamma Z}(0)}{m_Z^2} + \frac{\Sigma_Z(m_Z^2) - \Sigma_Z(0)}{m_Z^2} - \Sigma'_Z(m_Z^2);$$
$$\delta\rho_{f,v} = 2\frac{\delta a_f}{a_f} - 2\delta^v - \delta^b;$$
$$\delta\kappa_{f,v} = \frac{a_f\delta v_f - v_f\delta a_f}{-2Q_f a_f s_W^2}. \tag{2.8}$$

In above equations the subscript 'se' means the contribution from the gauge boson self-energy which is flavor independent, and 'f, v' denotes the contribution from the vertex correction to $Zf\bar{f}$ interaction. In practice, it is convenient to express $\delta\rho_{f,v}$ and $\delta\kappa_{f,v}$ in term of δg_L^f and δg_R^f respectively

$$\delta\rho_{f,v} = \frac{\delta g_L^f - \delta g_R^f}{a_f} - 2\delta^v - \delta^b; \qquad \delta\kappa_{f,v} = \frac{(a_f - v_f)\delta g_L^f + (a_f + v_f)\delta g_R^f}{-4Q_f a_f s_W^2} \qquad (2.9)$$

where $\delta g_{L,R}^f = \delta v_f \pm \delta a_f$ are the corrections to $Zf_L\bar{f}_L$ and $Zf_R\bar{f}_R$ interactions, respectively. From above equations one can learn that the correction to $\delta\rho_{f,v}$ is decided by the competition of δg_L^f and δg_R^f, while $\delta\kappa_{f,v}$ is mainly determined by δg_R^f due to $(a_f + v_f)/(a_f - v_f) \simeq 5.4$.

Noting that the Feynman rules for Z-boson couplings in SUSY models usually differ from their corresponding rules in the SM by a minus sign [8, 9], $\Sigma_{\gamma Z}$ and $\delta\kappa_{f,v}$ in the above formula should change sign if one uses the Feynman rules in SUSY models. The self-energies and the vertex corrections in SUSY models then include both the SM-particle loop contributions and SUSY-particle loop contributions. Since the SM-particle contributions are well known, in appendix A and B we only list the one-loop expressions for the SUSY contributions. The only subtlety one should note is to avoid the double-counting of the Higgs contributions. This problem arises due to the following reason. On the one hand, the SM values of ρ_b and $\sin^2\theta_{\text{eff}}^b$ are known to higher orders, and one usually incorporates such high-order SM effects when performing numerical calculations in SUSY models. On the other hand, because the SUSY Higgs sector is quite different from the SM, one cannot get the SUSY Higgs contributions simply by adding some additional terms to the SM Higgs contributions. In our calculation in SUSY models, to avoid the double-counting of the Higgs contributions, we first subtract the SM Higgs contributions from their SM values (calculated by the codes TOPAZ0 [16] and ZFITTER [17]), and then we add the full one-loop contributions from the SUSY Higgs bosons and sparticles.

3. Constraints on SUSY parameters

Before we proceed to discuss the SUSY corrections to $Zb\bar{b}$ coupling in the MSSM and NMSSM, we take a look at the SUSY parameters involved in our calculations. From the expressions of $Zf\bar{f}$ vertex correction listed in appendix B, one can learn that the SUSY-EW correction depends on the masses and the mixings of top squarks, bottom squarks, charginos and neutralinos, the SUSY-QCD vertex correction depends on gluino mass and the masses and the chiral mixing of bottom squarks, and the Higgs-mediated vertex correction depends on the masses and the mixings of Higgs bosons. The expressions of the gauge boson self-energies listed in appendix A indicate that the SUSY correction also depends on the masses of sleptons and the first-two generation squarks. About these SUSY parameters, we consider the following constraints

(1) Constraints from the direct search for the sparticles at LEP and Tevatron [18]

$$m_{\tilde{\chi}_1^0} > 41 \text{ GeV}, \quad m_{\tilde{\chi}_2^0} > 62.4 \text{ GeV}, \quad m_{\tilde{\chi}_3^0} > 99.9 \text{ GeV}, \quad m_{\tilde{\chi}^\pm} > 94 \text{ GeV},$$

$$m_{\tilde{e}} > 73 \text{ GeV}, \qquad m_{\tilde{\mu}} > 94 \text{ GeV}, \qquad m_{\tilde{\tau}} > 81.9 \text{GeV}, \qquad m_{\tilde{q}} > 250 \text{ GeV},$$

$$m_{\tilde{t}} > 89 \text{ GeV}, \qquad m_{\tilde{b}} > 95.7 \text{ GeV}, \qquad m_{\tilde{g}} > 195 \text{ GeV},$$

where $m_{\tilde{\chi}_i^0}$ denote the masses of the neutralinos and $m_{\tilde{q}}$ denotes the masses for the first two generation squarks.

(2) Constraint from the direct search for Higgs boson at LEP [19]. This constraint can limit the values of m_A, $\tan\beta$ and the masses and the chiral mixing of top squarks. In case of large $\tan\beta$, it can also put constraints on the masses and the mixing of bottom squarks. Generally speaking, this constraint requires the product of two top squark masses, $m_{\tilde{t}_1} m_{\tilde{t}_2}$, should be much larger than m_t^2 [20].

(3) Constraint from the theoretical requirements that there is no Landau pole for the running Yukawa couplings Y_b and Y_t below the GUT scale, and that the physical minimum of the Higgs potential with non-vanishing $\langle H_u \rangle$ and $\langle H_d \rangle$ is lower than the local minima with vanishing $\langle H_u \rangle$ and $\langle H_d \rangle$.

(4) Constraints from precision electroweak observalbes such as ρ_{lept}, $\sin^2\theta_{\text{eff}}^{\text{lept}}$, ρ_c, $\sin^2\theta_{\text{eff}}^c$ and M_W. These constraints are equivalent to those from the well known $\epsilon_i (i = 1, 2, 3)$ parameters [23] or S, T and U parameters [24]. The measured values of these observables are [1]

$$\rho_{\text{lept}} = 1.0050 \pm 0.0010, \quad \sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23153 \pm 0.00016,$$
$$\rho_c = 1.013 \pm 0.021, \qquad \sin^2\theta_{\text{eff}}^c = 0.2355 \pm 0.0059, \qquad M_W = 80.403 \pm 0.029 \text{ GeV},$$

and their SM fitted values are $\rho_{\text{lept}}^{SM} = 1.0051$, $\sin^2\theta_{\text{eff}}^{lept,SM} = 0.23149$, $\rho_c^{SM} = 1.0058$, $\sin^2\theta_{\text{eff}}^c = 0.2314$ and $M_W = 80.36 \text{ GeV}$ for $m_t = 173 \text{ GeV}$ and $m_h = 111 \text{ GeV}$. In our calculations we require the theoretical predictions to agree with the experimental values at 2σ level.

(5) Constraint from $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to hadrons)$. The measured value of R_b is $R_b^{\text{exp}} = 0.21629 \pm 0.00066$ and its SM prediction is $R_b^{SM} = 0.21578$ for $m_t = 173 \text{ GeV}$ [18]. In our analysis, we require R_b^{SUSY} is within the 2σ range of its experimental value.

(6) Constraint from the relic density of cosmic dark matter, i.e. $0.0945 < \Omega h^2 < 0.1287$ [21]. This constraint can rule out a broad parameter region for guagino masses $M_{1,2}$, μ parameter, m_A and $\tan\beta$ [22].

(7) Constraint from the muon anomalous magnetic momentum, a_μ. Now both the theoretical prediction and the experimental measurement of a_μ have reached a remarkable precision, which show a significant deviation $a_\mu^{\text{exp}} - a_\mu^{SM} = (29.5 \pm 8.8) \times 10^{-10}$ [25]. In our analysis we require the SUSY effects to account for such difference at 2σ level.

Note that in our analysis we do not include the constraints from B physics, like $b \to s\gamma$ [28] and $B_s - \bar{B}_s$ mixing [29], because these constraints are sensitive to squark flavor mixings which are irrelevant to our discussion.

Among the constraints listed above, the constraints (4) and (5), especially the observables M_W, ρ_{lept}, $\sin^2\theta_{\text{eff}}^{\text{lept}}$ and R_b, are most relevant to our study of ρ_b and $\sin^2\theta_{\text{eff}}^b$. Let us look at these constraints in more details.

First, the precise measurements of M_W, ρ_{lept} and $\sin^2\theta_{\text{eff}}^{\text{lept}}$ stringently constrain $\delta\rho_{se}$, $\delta\kappa_{se}$ and the gaugino loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$. The approximate forms of the SUSY corrections to M_W, $\delta\rho_{se}$ and $\delta\kappa_{se}$ [26] in case of heavy sparticles are given by

$$
\begin{aligned}
\frac{\delta M_W}{M_W} &= \frac{s_W^2}{c_W^2 - s_W^2}\frac{\delta(\Delta r)}{2(1 - \Delta r)} \simeq -\frac{c_W^2}{c_W^2 - s_W^2}\frac{\Delta\rho}{2}, \\
\delta\rho_{se} &\simeq \Delta\rho, \\
\delta\kappa_{se} &\simeq \frac{c_W^2}{s_W^2}\Delta\rho,
\end{aligned}
\tag{3.1}
$$

where

$$
\Delta\rho = \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2} - 2\frac{\sin\theta_W}{\cos\theta_W}\frac{\Sigma_{\gamma Z}(0)}{m_Z^2}
\tag{3.2}
$$

is the correction to the classical ρ parameter [6] and is only sensitive to the mass spectrum of the third generation squarks. Through the above relations the precisely measured M_W then stringently restricts $\Delta\rho$ (of order $O(10^{-4})$) and subsequently restricts $\delta\rho_{se}$ and $\delta\kappa_{se}$. This restriction together with the precisely determined ρ_{lept} and $\sin^2\theta_{\text{eff}}^{\text{lept}}$ stringently constrains the magnitude of $\delta\rho_{l,v}$ and $\delta\kappa_{l,v}$ defined in eq. (2.8) to be below $O(10^{-4})$. Since the gaugino loop effects in $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ are strongly correlated with $\delta\rho_{l,v}$ and $\delta\kappa_{l,v}$ (the main difference is caused by the mass difference between sleptons and squarks), the gaugino loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ are also suppressed, which are found to be below 5×10^{-4} from our numerical calculations.

For the constraint from the precision observable R_b, an interesting character is that it does not stringently constrain the magnitude of δv_b and δa_b, but it favors the relation $\delta v_b \sim -1.44\delta a_b$, which can be seen from the expression of the radiative correction to R_b [10–12]

$$
\delta R_b \simeq \frac{2R_b^{SM}(1 - R_b^{SM})}{v_b^2(3 - \beta^2) + 2a_b^2\beta^2}\left[v_b(3 - \beta^2)\delta v_b + 2a_b\beta^2\delta a_b\right] \propto (\delta v_b + 1.44\delta a_b)
\tag{3.3}
$$

with $\beta = \sqrt{1 - m_b^2/m_Z^2}$ being the velocity of bottom quark in Z decay.

Now we turn to the constraint from the muon anomalous magnetic momentum. To get an intuitive understanding of this constraint, we look at a simple case of the MSSM that all the gaugino masses and soft-breaking masses in smuon sector have a common scale M. In this case, a_μ^{SUSY} is approximated by [27]

$$
a_\mu^{\text{SUSY}} \simeq 13 \times 10^{-10}\left(\frac{100\text{ GeV}}{M}\right)^2 \tan\beta\ sign(\mu).
\tag{3.4}
$$

The gap between a_μ^{SM} and a_μ^{exp} then prefers a positive μ, and constrains the product $\left(\frac{100\text{ GeV}}{M}\right)^2\tan\beta$ in the range [1.0,3.6] at 2σ level. So the SUSY scale can be higher for larger $\tan\beta$.

In our calculations we use the code NMSSMTools [30] to generate the masses and the mixings for all sparticles and Higgs bosons in the framework of the NMSSM with all known radiative corrections included. There are two advantages in using this code. One is that all the masses and the mixings in the MSSM can be easily recovered if we set the parameters $\lambda = \kappa \simeq 0$ and A_κ to be negatively small. The other is that it incorporates the code MicrOMEGAs [31] which calculates the relic density of cosmic dark matter. It should be noted that the current version of NMSSMTools only includes the constraints (1), (2), (3) and (6), and we extend it by including the constraints (4), (5) and (7). We note that the muon anomalous magnetic momentum was recently calculated in the NMSSM [32] and our calculations agree with theirs.

4. One-loop corrections to ρ_b and $\sin^2 \theta_{\text{eff}}^b$ in MSSM

In this section we investigate ρ_b and $\sin^2 \theta_{\text{eff}}^b$ to one-loop level in the MSSM. As discussed above, the self-energy corrections to these two observables are generally small and thus we mainly scrutinize the vertex corrections which include the SUSY-EW corrections, the SUSY-QCD corrections and the Higgs-mediated vertex corrections. We pay special attention to the cases where the magnitudes of the corrections are large, and show that $\tan \beta$ is crucial in enhancing the vertex corrections. Our analysis is organized as follows: we first investigate the characters of the vertex corrections to get an intuitive understanding of them, then by scanning over the MSSM parameter space, we study the compatibility of the MSSM predictions for ρ_b and $\sin^2 \theta_{\text{eff}}^b$ with their experimental results.

The SM input parameters involved in our calculations are taken from [18], which are $\alpha = 1./128.93$, $G_F = 1.16637 \times 10^{-5}$, $\alpha_s(m_Z) = 0.1172$, $m_Z = 91.1876\,\text{GeV}$, $m_b(m_b) = 4.2\,\text{GeV}$ and $m_t = 172.5\,\text{GeV}$.

4.1 Characters of vertex corrections in MSSM

As for the SUSY-EW contribution to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$, the parameters involved are guagino masses $M_{1,2}$, Higgsino mass μ, $\tan \beta = v_2/v_1$ with $v_{1,2}$ being the vacuum expectation values of the Higgs fields, the soft-breaking masses M_{Q_3}, M_{U_3}, M_{D_3}, and the coefficients of the trilinear terms A_t and A_b. The first four parameters enter the mass matrices of neutralinos and charginos, and the last seven parameters affect the masses and the chiral mixings of the third generation squarks [8].

As discussed in the preceding section, the gaugino loop contribution is small, and hence we only discuss the Higgsino loop contribution. The magnitude of such Higgsino loop contribution is sensitive to $\tan \beta$, the Higgsino mass μ, and the masses and the chiral mixings of the third generation squarks. There are two characters for this contribution. One is that, due to the fact that the bottom Yukawa coupling Y_b is proportional to $1/\cos \beta$, the contribution can be potentially large in case of large $\tan \beta$ and small μ. The other is that the contribution is moderately sensitive to the chiral mixings of the third generation squarks, and potentially large contribution comes from the case where the mixing is small and the component of the lighter squark is dominated by the left-handed squark [11]. To illustrate these characters we consider three cases in the squark sector:
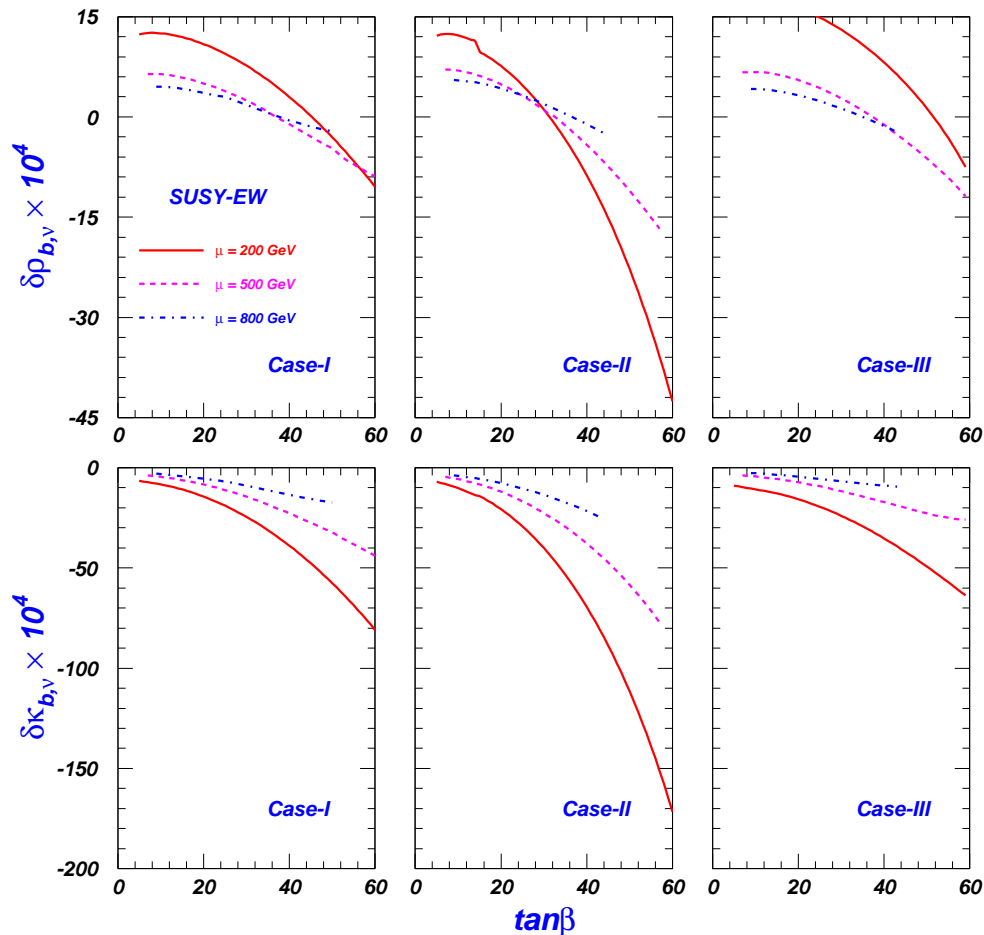
Figure 1: SUSY-EW contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ with constraints (1-5).

(I) $M_S = M_{Q_3} = M_{U_3} = M_{D_3} = 400\,\text{GeV}$, $A_t = A_b = 800\,\text{GeV}$;

(II) $M_{Q_3} = 200\,\text{GeV}$, $M_{U_3} = M_{D_3} = 600\,\text{GeV}$, $A_t = A_b = 800\,\text{GeV}$;

(III) $M_{Q_3} = 600\,\text{GeV}$, $M_{U_3} = M_{D_3} = 200\,\text{GeV}$, $A_t = A_b = 800\,\text{GeV}$,

and fix other SUSY parameters as

$$M_1 = 75 \text{ GeV}, \quad M_2 = 150 \text{ GeV}, \quad m_A = 500 \text{ GeV}, \quad M_{\text{SUSY}} = 1 \text{ TeV}, \tag{4.1}$$

where M_{SUSY} denotes the soft-breaking parameters for sleptons and the first-two generation squarks. Case-I corresponds to maximal chiral mixing case, Case-II is the small mixing case with the component of the lighter squark dominated by the left-handed squark and Case-III is also the small mixing case but with the component of the lighter squark dominated by the right-handed squark.

In figure 1 we show the dependence of the SUSY-EW contribution to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ on $\tan\beta$ in the three cases. One can see that both $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ are sensitive to $\tan\beta$. As $\tan\beta$ increases, $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ get more negative contributions and, for small μ, they become negative with sizable magnitudes. This behavior can be understood as following.

As $\tan\beta$ gets large, the bottom Yukawa coupling increases and the correction to the right-handed $Zb\bar{b}$ coupling δg_R^b increases positively, and then $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ get more negative contribution from the increasing δg_R^b(see eq. (2.9) and also δg_R^b in appendix B). One also see from these figures that the magnitude of $\delta\kappa_{b,v}$ is usually larger than $\delta\rho_{b,v}$. The factor $\sin^2\theta_W$ in the denominator of $\delta\kappa_{b,v}$ (see eq. (2.8)) can to a large extent account for this.

Note that in these figures we only plot our results within the range of $\tan\beta$ that survives the constraints (1-5). The constraint (7), i.e. the muon anomalous magnetic moment, can in principle also limit $\tan\beta$. But this constraint relies on the mass scale of smuon, $M_{\rm SUSY}$ in eq. (4.1), which ρ_b and $\sin^2\theta_{\rm eff}^b$ are not sensitive to, so we do not apply it in plotting these figures. Our numerical results indicate that the muon anomalous magnetic moment allows for a vast region of $M_{\rm SUSY}$ and μ where $\tan\beta$ can be as large as 60, and hence the sizable SUSY-EW corrections to ρ_b and $\sin^2\theta_{\rm eff}^b$ are possible. For example, with the parameters in eq. (4.1), the range of $\tan\beta$ allowed by the muon $g-2$ is $\tan\beta \geq 25$ for $\mu = 200\,{\rm GeV}$, $\tan\beta \geq 33$ for $\mu = 500\,{\rm GeV}$, and $\tan\beta \geq 44$ for $\mu = 800\,{\rm GeV}$. If we choose $M_{\rm SUSY} = 0.5\,{\rm TeV}$, these allowed ranges are correspondingly given by $7 \leq \tan\beta \leq 57$, $12 \leq \tan\beta \leq 71$ and $\tan\beta \geq 14$.

Next we discuss the SUSY-QCD corrections. The relevant parameters are gluino mass and M_{Q_3}, M_{D_3} and $X_b = (A_b - \mu\tan\beta)$ which enter the mass matrix of the bottom squarks. From the large strength of the strong coupling, $g_s(m_Z) \simeq 1.2 \simeq 50 \times Y_b^{SM}$, one may naively postulates that the SUSY-QCD contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ should be much larger than the Higgsino loop contributions in case of $m_{\tilde{g}} \simeq \mu$ and $\tan\beta \ll 50$. However, our numerical results show that in case of small sbottom chiral mixing the SUSY-QCD contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ are negligibly small. The underlying reason is that for the SUSY-QCD corrections there is a strong cancellation between different diagrams in case of small sbottom chiral mixing, which can be seen from the expressions of $\delta g_{L,R}^b$ listed in appendix B. It should be noted that such a cancellation can be alleviated for a large sbottom mixing, or equivalently, a large term $\mu\tan\beta$ appeared in the non-diagonal elements of sbottom mass matrix (we checked this from numerical calculations). So the contribution may be sizable in case of large $\mu\tan\beta$, as shown in figure 2.

Compared with the Higgsino loop corrections, the SUSY-QCD contributions in figure 2 exhibit a similar behavior with respect to $\tan\beta$. The difference is that the most sizable effects come from Case-I (maximal sbottom mixing case) with large μ, instead of Case-II with small μ for the Higgsino loop corrections.

Finally, we consider the Higgs loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ [33]. To calculate this part of contribution, we need to know the masses and the mixing of the Higgs bosons, which are determined by m_A and $\tan\beta$ at tree-level, and also by the soft-breaking masses for the third generation squarks if the important loop correction to the Higgs boson masses is taken into account. As shown in figure 3, the contributions exhibit a similar dependence on $\tan\beta$, and the significant contribution comes from the case of small m_A and large $\tan\beta$. We checked that the results in figure 3 are not sensitive to μ or M_S, and also not sensitive to the choice of different case (Case-I, Case-II or Case-III).

From the above figures one can infer that among the three types of corrections, the potentially largest correction comes from the Higgs loops, which can reach 2% for ρ_b and
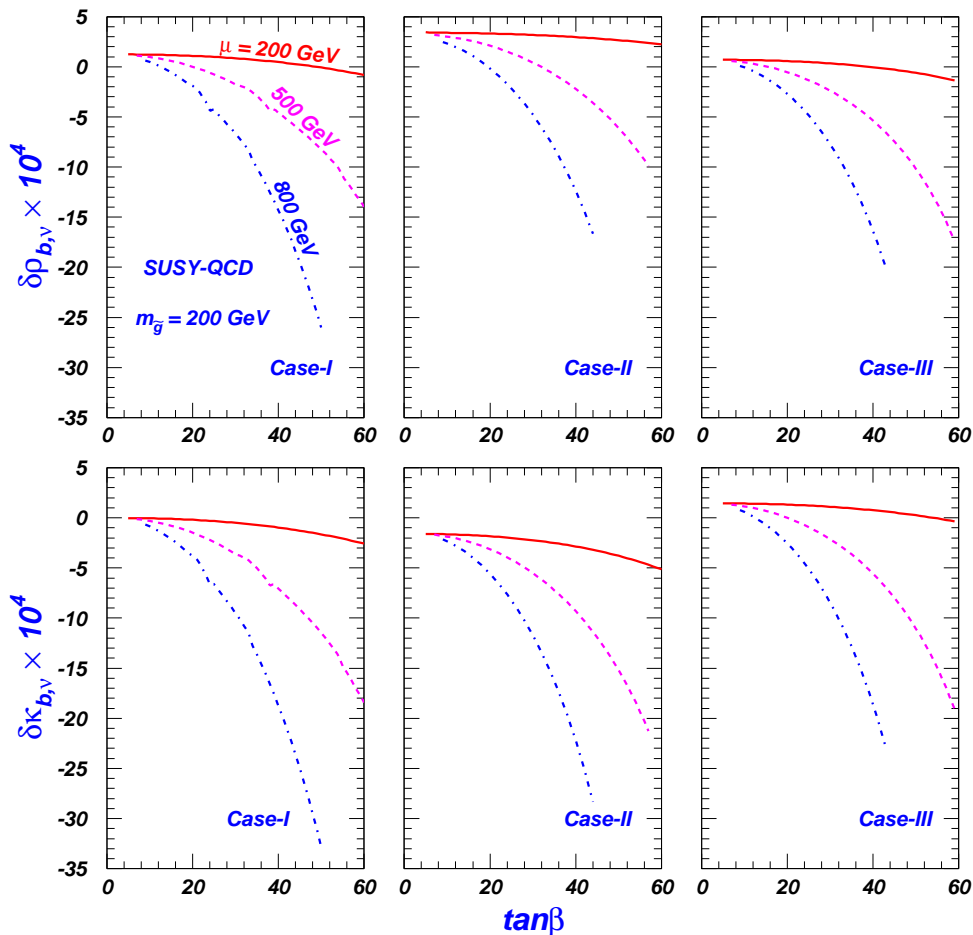
Figure 2: SUSY-QCD contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ with constraints (1-5).

6% for $\sin^2\theta_{\text{eff}}^b$. Such large corrections reach the current experimental sensitivity since the current experimental measurements are $\rho_b^{\text{exp}} = 1.059 \pm 0.021$ and $\sin^2\theta_{\text{eff}}^{b,exp} = 0.281 \pm 0.016$.

Before we end this section, we would like to point out that in the large $\tan\beta$ limit the relic density of cosmic dark matter allows the possibility of small μ or small m_A (but not both small). This can be seen from figure 4, where we show the allowed regions in the plane of $\tan\beta$ versus μ for different m_A. In plotting this figure, we choose Case-I and fix other related parameters in eq. (4.1). Figure 4 implies that the SUSY-EW contribution and the Higgs-loop contribution to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ cannot simultaneously reach their maximal values.

4.2 MSSM predictions for ρ_b and $\sin^2\theta_{\text{eff}}^b$

As mentioned above, the extracted values of ρ_b and $\sin^2\theta_{\text{eff}}^b$ from combined LEP and SLD data analysis are respectively 1.059 ± 0.021 and 0.281 ± 0.016 with correlation coefficient 0.99 [1]. This result is shown in figure 5 with the three ellipses corresponding to 68%, 95.5% and 99.5% confidence level (CL), respectively. Noting that the SM predictions are $\rho_b^{SM} = 0.994$ and $\sin^2\theta_{\text{eff}}^{bSM} = 0.233$, one may infer that large positive corrections to ρ_b
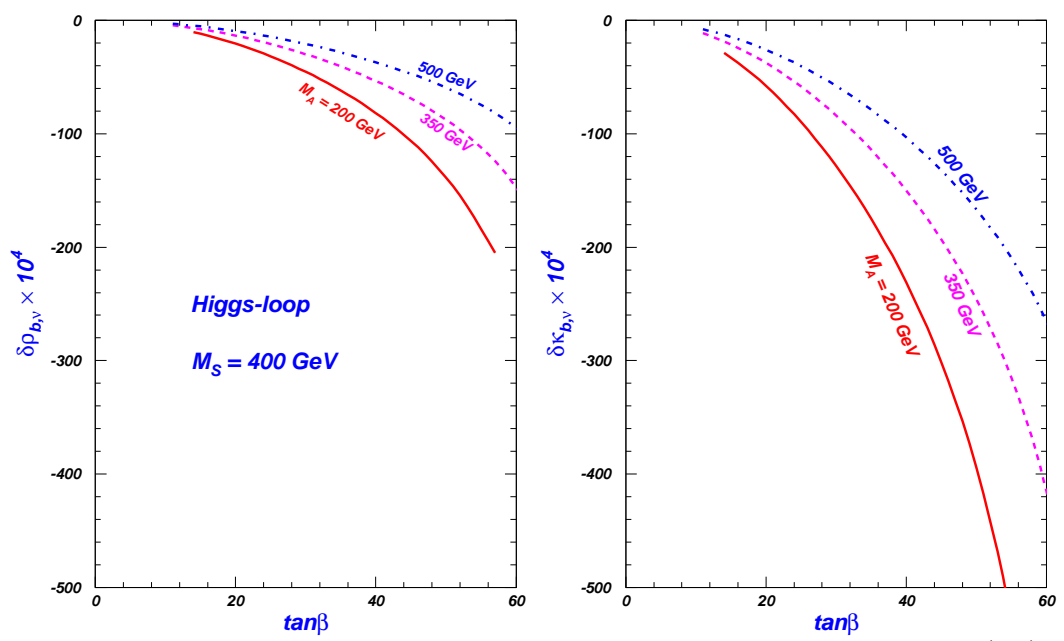
Figure 3: Higgs loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ in Case-I with constraints (1-5).
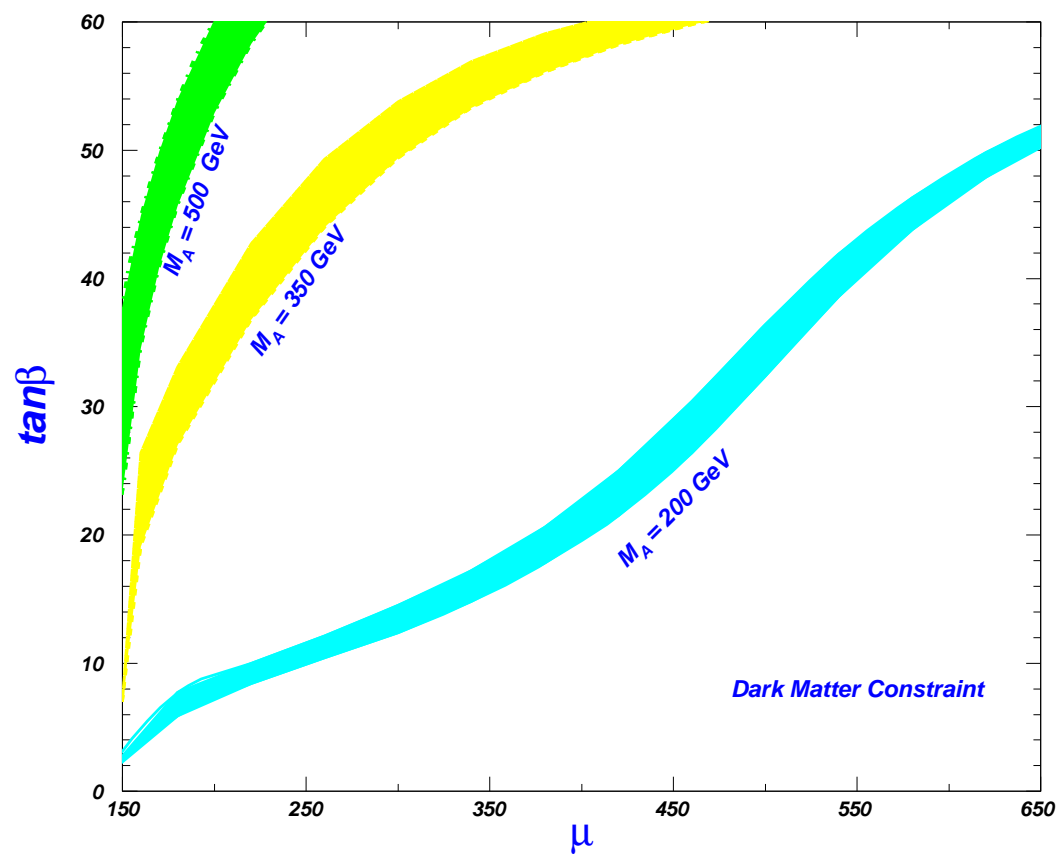

Figure 4: The shaded regions are allowed by the cosmic dark matter relic density at 2σ level. Other relevant SUSY parameters are fixed as in Case-I and in eq. (4.1).
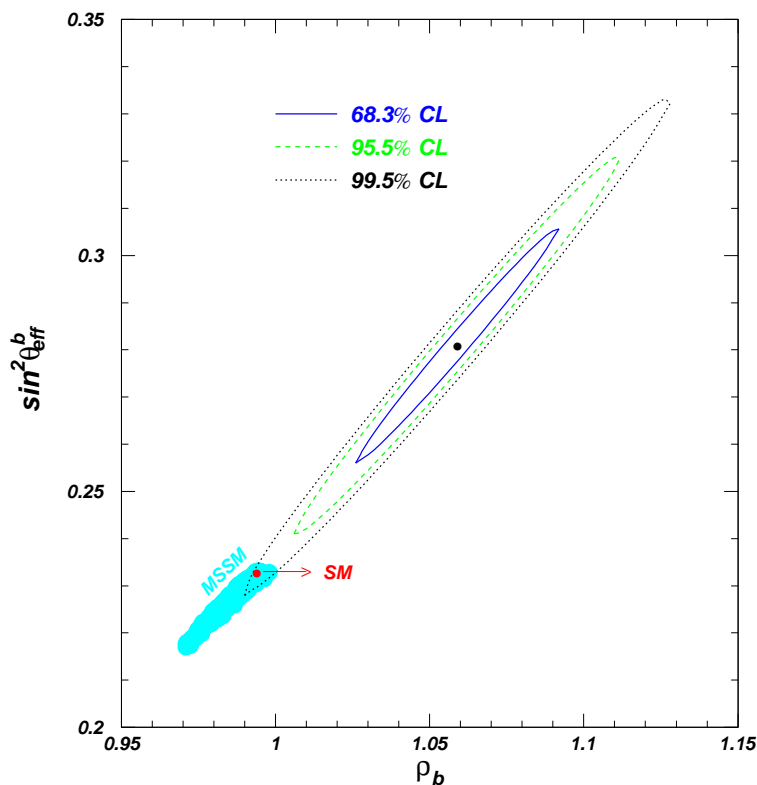
Figure 5: The MSSM and SM predictions for ρ_b and $\sin^2 \theta_{\text{eff}}^b$, compared with the LEP/SLD data at 68%, 95.5% and 99.5% confidence level. The SM prediction $\rho_b^{SM} = 0.994$ and $\sin^2 \theta_{\text{eff}}^{b,SM} = 0.233$ is obtained with $m_t = 174\,\text{GeV}$ and $m_h = 115\,\text{GeV}$. The MSSM predictions are from a scan (a sample of one million) over the parameter space.

and $\sin^2 \theta_{\text{eff}}^b$ are needed to narrow the gap between the experimental data and the SM prediction. As discussed in the preceding section, the MSSM corrections can be sizable for large $\tan \beta$, which, however, are negative and thus cannot narrow the gap. To figure out to what extent the MSSM predictions can agree with the experiment, we consider all the constraints discussed in section III and scan over the SUSY parameter space:

$$0 < M_1, M_2, M_3, \mu, M_{Q_3}, M_{U_3}, M_{D_3}, M_A, M_{\text{SUSY}} \leq 1 \text{ TeV},$$
$$-3 \text{ TeV} \leq A_t, A_b \leq 3 \text{ TeV}, \qquad 1 < \tan \beta \leq 60, \tag{4.2}$$

Based on a twenty billion sample, we find the best MSSM predictions are $\rho_b = 0.9960$ and $\sin^2 \theta_{\text{eff}}^b = 0.2328$, which give a $\chi^2/dof = 9.07/2$ when compared with the experiment data. If we do not consider the dark matter constraint, the best MSSM predictions are $\rho_b = 0.99737$ and $\sin^2 \theta_{\text{eff}}^b = 0.2336$, which give a $\chi^2/dof = 8.77/2$. Moreover, we find that such a best case happens when $\mu, m_A, m_{\tilde{g}} \sim 1\,\text{TeV}$ so that the three types of vertex corrections are suppressed.

5. One-loop predictions for ρ_b and $\sin^2\theta^b_{\text{eff}}$ in NMSSM

5.1 Introduction to the NMSSM

As a popular extension of the MSSM, the NMSSM provides an elegant solution to the μ-problem via introducing a singlet Higgs superfield $\hat{S}$, which naturally develops a vacuum expectation value of the order of the SUSY breaking scale and gives rise to the required μ term. Another virtue of the NMSSM is that it can alleviate the little hierarchy problem since the theoretical upper bound on the SM-like Higgs boson mass is pushed up and the LEP II lower bound on the Higgs boson mass is relaxed due to the suppressed ZZh coupling or the suppressed decay $h \to b\bar{b}$ [36]. Since the NMSSM is so well motivated, its phenomenology has been intensively studied in recent years, such as its effects in Higgs physics [37], neutralino physics [38], B-physics [39] as well as squark physics [40]. In the following we recapitulate the basics of the NMSSM with emphasis on its difference from the MSSM.

The superpotential of the NMSSM takes the form [9, 30]

$$W = \lambda\varepsilon_{ij}\hat{H}^i_u\hat{H}^j_d\hat{S} + \frac{1}{3}\kappa\hat{S}^3 + h_u\varepsilon_{ij}\hat{Q}^i\hat{U}\hat{H}^j_u - h_d\varepsilon_{ij}\hat{Q}^i\hat{D}\hat{H}^j_d - h_e\varepsilon_{ij}\hat{L}^i\hat{E}\hat{H}^j_d \qquad (5.1)$$

where $\hat{S}$ is the singlet Higgs superfield, and $\varepsilon_{12} = -\varepsilon_{21} = 1$. For the soft SUSY breaking terms, we take

$$\begin{aligned}
V_{\text{soft}} = &\frac{1}{2}M_2\lambda^a\lambda^a + \frac{1}{2}M_1\lambda'\lambda' + m^2_d|H_d|^2 + m^2_u|H_u|^2 + m^2_S|S|^2 \\
&+ m^2_Q|\tilde{Q}|^2 + m^2_U|\tilde{U}|^2 + m^2_D|\tilde{D}|^2 + m^2_L|\tilde{L}|^2 + m^2_E|\tilde{E}|^2 \\
&+ (\lambda A_\lambda\varepsilon_{ij}H^i_uH^j_dS + \text{h.c.}) + \left(\frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}\right) \\
&+ (h_uA_U\varepsilon_{ij}\tilde{Q}^i\tilde{U}H^j_u - h_dA_D\varepsilon_{ij}\tilde{Q}^i\tilde{D}H^j_d - h_eA_E\varepsilon_{ij}\tilde{L}^i\tilde{E}H^j_d + \text{h.c.}) \qquad (5.2)
\end{aligned}$$

With the above configuration of the model, the μ parameter is given by $\mu = \lambda\langle S\rangle$ with $\langle S\rangle$ being the vacuum expectation value of S field, and the m_A parameter in the MSSM corresponds to the combination $m^2_A = \frac{2\mu}{\sin 2\beta}(A_\lambda + \frac{\kappa\mu}{\lambda})$ (see eq. (5.4)). So compared with the MSSM, the NMSSM has three additional input parameters λ, κ and A_κ. These three parameters should be subject to the constraints listed in section III, and the argument that the NMSSM should keep perturbative up to the Planck scale requires λ and κ to be smaller than 0.7.

The differences of the NMSSM and MSSM come from the Higgs sector and the neutralino sector. In the Higgs sector, now we have three CP-even and two CP-odd Higgs bosons. In the basis $[Re(H^0_u), Re(H^0_d), Re(S)]$, the mass-squared matrix entries for CP-even Higgs bosons are [9, 30]

$$\begin{aligned}
\mathcal{M}^2_{S,11} &= m^2_A\cos^2\beta + m^2_Z\sin^2\beta, \\
\mathcal{M}^2_{S,22} &= m^2_A\sin^2\beta + m^2_Z\cos^2\beta, \\
\mathcal{M}^2_{S,33} &= \frac{\lambda^2v^2}{4\mu^2}m^2_A\sin^2 2\beta - \frac{\lambda\kappa}{2}v^2\sin 2\beta + \frac{\kappa}{\lambda^2}\mu(\lambda A_\kappa + 4\kappa\mu), \\
\mathcal{M}^2_{S,12} &= (2\lambda^2v^2 - m^2_Z - m^2_A)\sin\beta\cos\beta,
\end{aligned}$$

$$\mathcal{M}^2_{S,13} = 2\lambda\mu v \sin\beta - \frac{\lambda v}{2\mu}m^2_A \sin 2\beta \cos\beta - \kappa\mu v \cos\beta,$$

$$\mathcal{M}^2_{S,23} = 2\lambda\mu v \cos\beta - \frac{\lambda v}{2\mu}m^2_A \sin\beta \sin 2\beta - \kappa\mu v \sin\beta, \tag{5.3}$$

and for the CP-odd Higgs bosons, their mass-squared matrix entries in the basis $[\tilde{A}, Im(S)]$ with $\tilde{A} = \cos\beta\, Im(H^0_u) + \sin\beta\, Im(H^0_d)$ are

$$\mathcal{M}^2_{P,11} = \frac{2\mu}{\sin 2\beta}\left(A_\lambda + \frac{\kappa\mu}{\lambda}\right) \equiv m^2_A,$$

$$\mathcal{M}^2_{P,22} = \frac{3}{2}\lambda\kappa v^2 \sin 2\beta + \frac{\lambda^2 v^2}{4\mu^2}m^2_A \sin^2 2\beta - 3\frac{\kappa}{\lambda}\mu A_\kappa,$$

$$\mathcal{M}^2_{P,12} = \frac{\lambda v}{2\mu}m^2_A \sin 2\beta - 3\kappa\mu v. \tag{5.4}$$

Eqs. (5.3) and (5.4) indicate that the parameters λ and $\kappa\mu$ affect the mixings of the doublet fields with the singlet field, A_κ only affects the squared-mass of the singlet field, and in the limit $\lambda, \kappa \to 0$, the NMSSM can recover the MSSM. One can also learn that in case of small λ and κ so that the mixings are small, the physical state with the singlet being the dominant component should couple weakly to bottom quarks and thus its loop contribution to ρ_b and $\sin^2\theta^b_{\text{eff}}$ should be small.

The NMSSM predicts five neutralinos, and in the basis $(-i\lambda_1, -i\lambda_2, \psi^0_u, \psi^0_d, \psi_s)$ their mass matrix is given by [9, 30]

$$\begin{pmatrix} M_1 & 0 & m_Z \sin\theta_W \sin\beta & -m_Z \sin\theta_W \cos\beta & 0 \\ & M_2 & -m_Z \cos\theta_W \sin\beta & m_Z \cos\theta_W \cos\beta & 0 \\ & & 0 & -\mu & -\lambda v \cos\beta \\ & & & 0 & -\lambda v \sin\beta \\ & & & & 2\frac{\kappa}{\lambda}\mu \end{pmatrix}. \tag{5.5}$$

This mass matrix is independent of A_κ, and the role of λ is to introduce the mixings of ψ_s with ψ^0_u and ψ^0_d, and $k\mu$ is to affect the mass of ψ_s. Quite similar to the discussion about the Higgs bosons, in case of small λ, the correction to ρ_b and $\sin^2\theta^b_{\text{eff}}$ should be insensitive to the value of $\kappa\mu$.

5.2 NMSSM correction to ρ_b and $\sin^2\theta^b_{\text{eff}}$

We first look at the SUSY-EW corrections in the NMSSM. Compared with the corresponding MSSM corrections, the NMSSM effects involve two additional parameters λ and κ. As discussed below eq. (5.5), in case of small λ, the corrections are insensitive to κ (our numerical results verified this conclusion), and thus here we mainly study the dependence on λ. We choose a value for κ so that the allowed range of λ is wide.

In figure 6 we show the SUSY-EW contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ as a function of λ, in which $\tan\beta = 40$, $\kappa = 0.4$, $A_\kappa = -100\,\text{GeV}$ and other parameters are same as in figure 1. One character of this figure is that both $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ become more negative with the increase of λ, which enlarges the gap between the theoretical values and the experimental
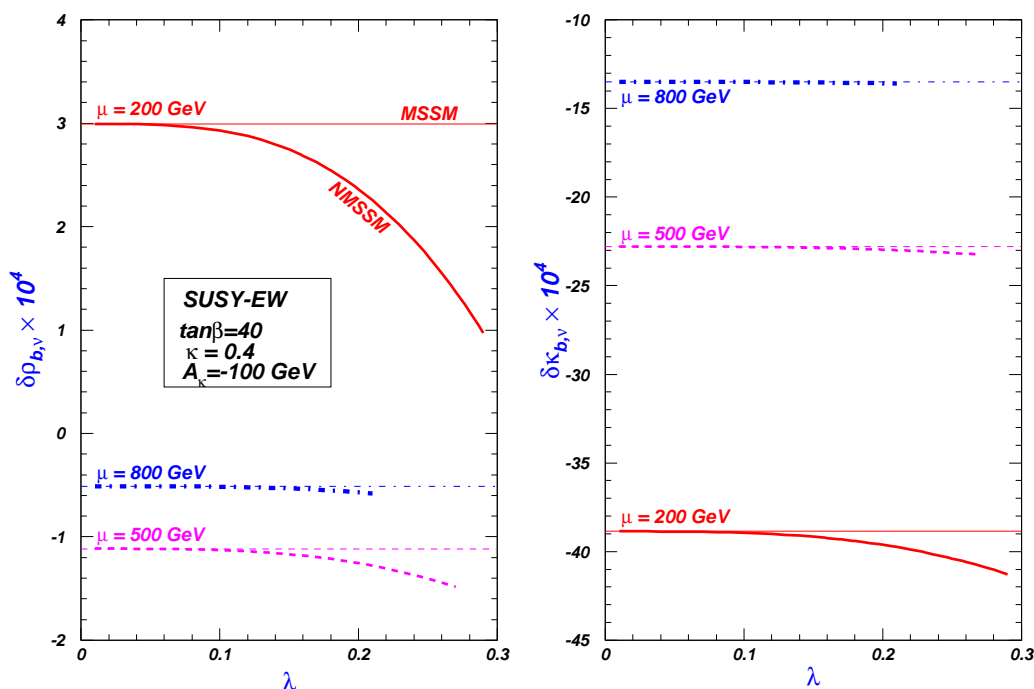
Figure 6: The NMSSM electroweak contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$, compared with the corresponding MSSM contributions (thin horizontal lines), under constraints (1-5).

data. Another character of this figure is that the contributions are less sensitive to λ when μ becomes large. This can be explained from eq. (5.5) which shows that the mixings between ψ_s and the doublets (ψ_u^0, ψ_d^0) become negligiblly small for sufficiently large μ and thus reduce the sensitivity of the contributions to λ.

We now turn to the Higgs loop contributions to $\delta\rho_{b,v}$ and $\delta\kappa_{b,v}$ in the NMSSM. For these contributions, besides m_A and $\tan\beta$, the parameters λ, κ and A_κ are also involved. Noting that these contributions are more sensitive to λ and κ than to A_κ, we only study their dependence on λ and κ.

In figure 7 we show the contributions versus λ, where $\tan\beta = 40$, $\kappa = 0.4$, $A_\kappa = -100\,\text{GeV}$ and other parameters are same as in figure 3. This figure shows the same behavior as in figure 6, and the dependence on λ becomes rather weak in case of large m_A.

In figure 8, we show the dependence of the contributions on κ, as shown. This figure exhibits the similar behavior to figure 7. Compared with figure 7 and figure 8, one can learn that the contributions have a stronger dependence on λ than on κ.

Like in figure 5, we also investigate the extent to which the NMSSM predictions can agree with the experiment by scanning over the SUSY parameter space in the region of eq. (4.2) and

$$\lambda, \kappa \leq 0.7, \qquad -1\,\text{TeV} < A_\kappa < 1\,\text{TeV}. \tag{5.6}$$

Our result is shown in figure 9. Compared with figure 5, one can learn that the NMSSM cannot improve the agreement and instead may exacerbate the agreement in a large part of the allowed parameter space.
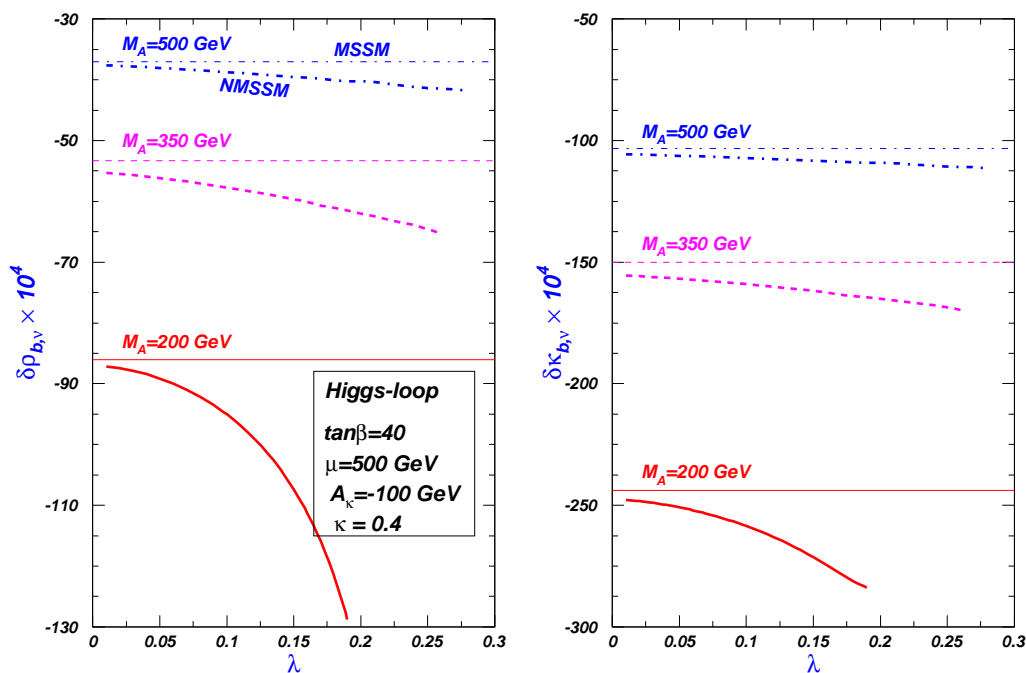
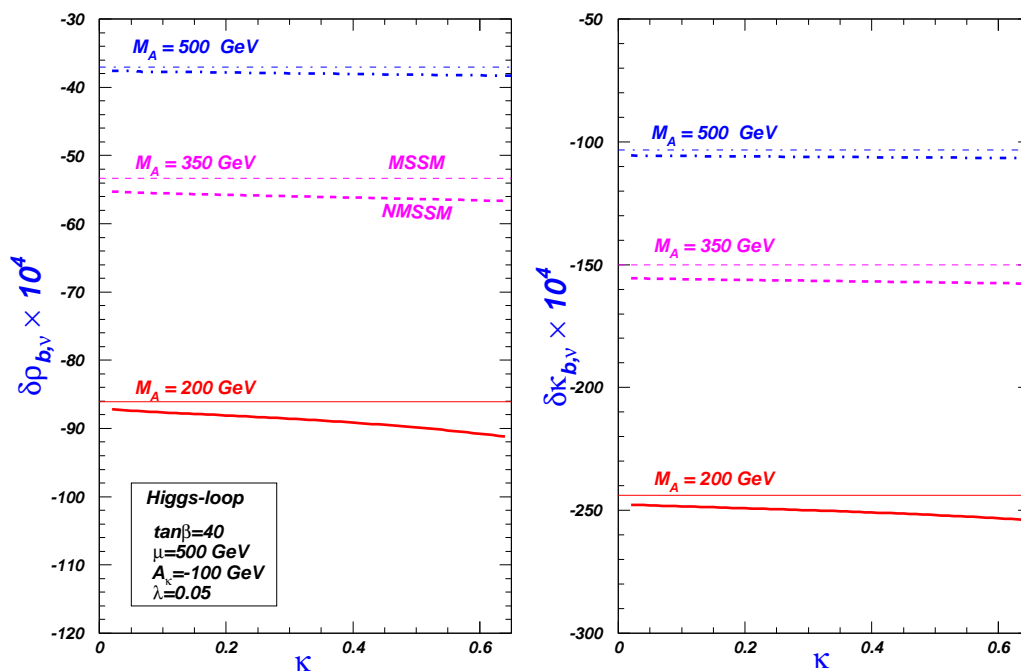**Figure 7:** Same as figure 6, but for the Higgs loop contributions.

Figure 8: Same as figure 6, but for the Higgs loop contributions versus the parameter κ.

If we define a quantity $F(\lambda, \kappa) - F(0, 0)$ with F denoting either $\delta\rho_{b,v}$ or $\delta\kappa_{b,v}$ with $F(\lambda, \kappa)$ being the value of F in the NMSSM with arbitrary values of λ and κ, and $F(0, 0)$ being the value of F in the MSSM limit, then by studying various cases we find this quantity
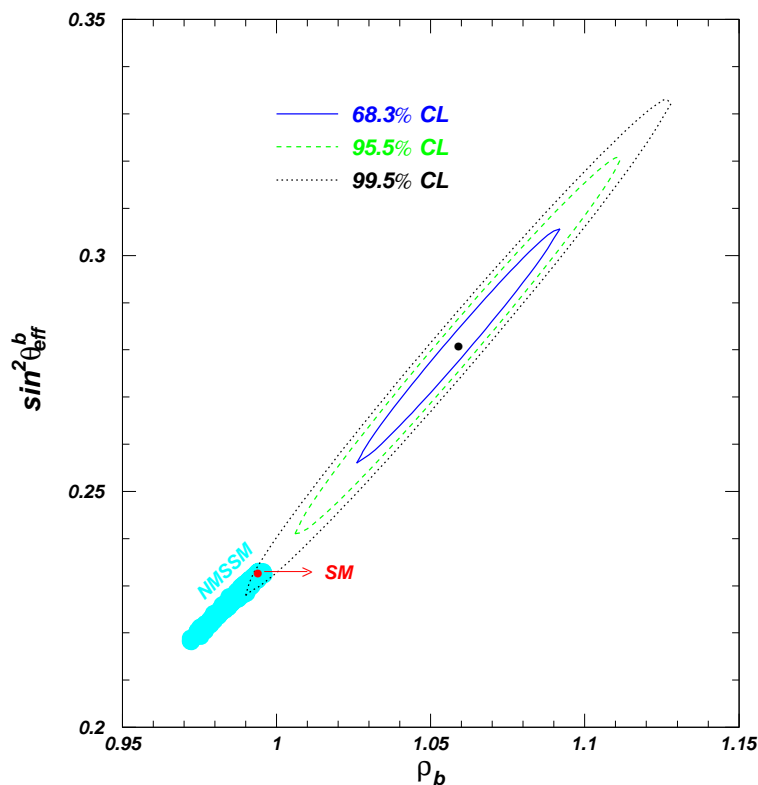
Figure 9: Same as figure 5, but for the NMSSM predictions.

is generally smaller than 5×10^{-3}, which means that in the allowed region for λ and κ, NMSSM only slightly modifies the MSSM predictions of ρ_b and $\sin^2 \theta_{\text{eff}}^b$.

6. Conclusions

The $Zb\bar{b}$ coupling determined from the Z-pole measurements at LEP/SLD deviate significantly from the SM prediction. In terms of ρ_b and $\sin^2 \theta_{\text{eff}}^b$, the SM prediction is about 3σ below the experimental data. If this anomaly is not a statistical or systematic effect, it would signal the presence of new physics in association with the $Zb\bar{b}$ coupling. In this work we scrutinized the full one-loop supersymmetric effects on $Zb\bar{b}$ coupling in both the MSSM and the NMSSM, considering all current constraints which are from the precision electroweak measurements, the direct search for sparticles and Higgs bosons, the stability of Higgs potential, the dark matter relic density, and the muon g-2 measurement. We analyzed the characters of each type of the corrections and searched for the SUSY parameter regions where the corrections could be sizable. We found that the potentially sizable corrections come from the Higgs sector with light m_A and large $\tan\beta$, which can reach -2% and -6% for ρ_b and $\sin^2 \theta_{\text{eff}}^b$, respectively. However, such sizable negative corrections are just opposite to what needed to solve the anomaly. We also scanned over the allowed parameter space and investigated to what extent supersymmetry can narrow the discrepancy between theoretical predictions and the experimental values. We found that

under all current constraints, the supersymmetric effects are quite restrained and cannot significantly ameliorate the anomaly of $Zb\bar{b}$ coupling. Compared with $\chi^2/dof = 9.62/2$ in the SM, the MSSM and NMSSM can only improve it to $\chi^2/dof = 8.77/2$ in the allowed parameter space.

In the future the GigaZ option at the proposed International Linear Collider (ILC) with an integrated luminosity of $30\,\text{fb}^{-1}$ is expected to produce more than 10^9 Z-bosons [41] and will give a more precise measurement of $Zb\bar{b}$ coupling, which will allow for a test of new physics models. If the anomaly of $Zb\bar{b}$ coupling persists, it would suggest new physics beyond the MSSM and NMSSM. One possible form of such new physics is the model with additional right-handed gauge bosons which couple predominantly to the third generation quarks [42]. These new gauge bosons usually mix with Z and W so that the $Zb_R\bar{b}_R$ and $Wb_R\bar{t}_R$ couplings in the SM may be greatly changed. A careful investigation of top quark processes at the LHC, such as top quark decay to the polarized W boson [43], may test this model in the near future.

Acknowledgments

A. Gauge boson self-energy in NMSSM

In the NMSSM the contributions to vector boson self-energy come from the loops mediated by the SM fermions, gauge bosons, Higgs bosons, sfermions, charginos and neutralinos, respetively. In the following we list the expressions for pure new physics contributions, namely from the loops of Higgs bosons, sfermions, charginos and neutralinos, respectively. We adopt the convention of [30] for the SUSY parameters.

(1) Higgs contribution:

The NMSSM has an extended Higgs boson sector with a pair of charged Higgs bosons $H^\pm$, two CP-odd Higgs boson a_i and three CP-even Higgs boson h_i. The Higgs contribution to gauge boson self-energy arises from VHH, $VVHH$ and VVH interactions and because we choose 't Hooft-Feynman gauge to calculate the contribution, the gauge boson contribution and the Higgs contribution are in general entangled. In our calculation, we are actually interested in the difference between the contribution from the NMSSM Higgs sector and that from the SM Higgs sector (see the discussion in the last paragraph of section II). Since the SM contribution is well known [14, 15], we only list the NMSSM contribution.

$$\Sigma_{\gamma\gamma}^T(p^2) = \frac{e^2}{16\pi^2} B_5(p, m_{H^+}, m_{H^+}), \tag{A.1}$$

$$\Sigma_{\gamma Z}^T(p^2) = \frac{1}{16\pi^2} \frac{eg\cos 2\theta_W}{2\cos\theta_W} B_5(p, m_{H^+}, m_{H^+}), \tag{A.2}$$

$$\Sigma_{ZZ}^T(p^2) = \frac{1}{16\pi^2}\frac{g^2}{4\cos^2\theta_W}\Bigg\{\Bigg[(|S_{i1}|^2 + |S_{i2}|^2)A(m_{h_i}) + |P'_{i1}|^2 A(m_{a_i}) + A(m_Z)$$
$$-4|\sin\beta S_{i2} - \cos\beta S_{i1}|^2|P'_{j1}|^2 B_{22}(p, m_{a_j}, m_{h_i})$$
$$-4|\cos\beta S_{i2} + \sin\beta S_{i1}|^2 B_{22}(p, m_Z, m_{h_i})\Bigg]$$
$$+2\cos^2 2\theta_W\Bigg[A(m_{H^+}) - 2B_{22}(p, m_{H^+}, m_{H^+})\Bigg]$$
$$+4m_Z^2|\cos\beta S_{i2} + \sin\beta S_{i1}|^2 B_0(p, m_Z, m_{h_i})\Bigg\}, \quad \text{(A.3)}$$

$$\Sigma_{WW}^T(p^2) = \frac{1}{16\pi^2}\frac{g^2}{4}\Bigg\{\Bigg[A(m_{H^+}) + (|S_{i1}|^2 + |S_{i2}|^2)A(m_{h_i}) + A(m_W)$$
$$-4|\sin\beta S_{i2} - \cos\beta S_{i1}|^2 B_{22}(p, m_{H^+}, m_{h_i})$$
$$-4|\cos\beta S_{i2} + \sin\beta S_{i1}|^2 B_{22}(p, m_W, m_{h_i})\Bigg]$$
$$+\Bigg[A(m_{H^+}) + |P'_{i1}|^2 A(m_{a_i}) - 4|P'_{i1}|^2 B_{22}(p, m_{H^+}, m_{a_i})\Bigg]$$
$$+4m_W^2|\cos\beta S_{i2} + \sin\beta S_{i1}|^2 B_0(p, m_W, m_{h_i})\Bigg\}, \quad \text{(A.4)}$$

In above equations, g is the SU(2) gauge coupling, and S and P' are the rotation mass matrices defined in the appendix A of [30] to diagonalize CP-even and CP-odd Higgs mass matrices, respectively. A and B_{22} are the standard one- and two-point loop functions firstly defined in [34]. B_5 is related with standard loop functions by [35]

$$B_5(p, m_1, m_2) = A(m_1) + A(m_2) - 4B_{22}(p, m_1, m_2). \quad \text{(A.5)}$$

(2) Sfermion contribution:

The sfermion contributions are given by

$$\Sigma_{WW}^T(p^2) = \frac{1}{16\pi^2}\frac{g^2}{2}C_f R_{\alpha 1}^{\tilde{u}*} R_{\alpha 1}^{\tilde{u}} R_{\beta 1}^{\tilde{d}*} R_{\beta 1}^{\tilde{d}} B_5(p, m_{\tilde{u}_\alpha}, m_{\tilde{d}_\beta}), \quad \text{(A.6)}$$

$$\Sigma_{ZZ}^T(p^2) = \frac{1}{16\pi^2}\frac{g^2}{\cos^2\theta_W}C_f\Bigg\{I_{3f}^2 R_{\alpha 1}^{\tilde{f}*} R_{\alpha 1}^{\tilde{f}} R_{\beta 1}^{\tilde{f}*} R_{\beta 1}^{\tilde{f}} B_5(p, m_{\tilde{f}_\alpha}, m_{\tilde{f}_\beta})$$
$$-2s_W^2 I_{3f} Q_f R_{\alpha 1}^{\tilde{f}*} R_{\alpha 1}^{\tilde{f}} B_5(p, m_{\tilde{f}_\alpha}, m_{\tilde{f}_\alpha}) + s_W^4 Q_f^2 B_5(p, m_{\tilde{f}_\alpha}, m_{\tilde{f}_\alpha})\Bigg\}, \text{(A.7)}$$

$$\Sigma_{\gamma\gamma}^T(p^2) = \frac{e^2}{16\pi^2}C_f Q_f^2 B_5(p, m_{\tilde{f}_\alpha}, m_{\tilde{f}_\alpha}), \quad \text{(A.8)}$$

$$\Sigma_{\gamma Z}^T(p^2) = \frac{e}{16\pi^2}\frac{g}{\cos\theta_W}C_f\Bigg\{I_{3f}Q_f R_{\alpha 1}^{\tilde{f}*} R_{\alpha 1}^{\tilde{f}} - Q_f^2 s_W^2\Bigg\}B_5(p, m_{\tilde{f}_\alpha}, m_{\tilde{f}_\alpha}), \quad \text{(A.9)}$$

where the color factor C_f is 3 for squarks and 1 for sleptons. The electric charge Q_f is given by $2/3, -1/3, 0, -1$ for $\tilde{u}, \tilde{d}, \tilde{\nu}_l, \tilde{l}$, respectively. I_{3f} denotes the third component of the weak isospin, which is $+1/2$ and $-1/2$ for the up- and down-type sfermions, respectively. R is the rotation matrix to diagonalize sfermion mass matrix.

(3) Chargino and neutralino contribution:

For a generic interaction between a vector boson and two fermions, it contributes to vector boson self-energy in the form:

$$\Sigma^T_{V'V}(p^2) = \frac{2}{16\pi^2}\Bigg\{(g_L^{\bar{\psi}_j\psi_iV'}g_L^{\bar{\psi}_i\psi_jV^*} + g_R^{\bar{\psi}_j\psi_iV'}g_R^{\bar{\psi}_i\psi_jV^*})(2p^2B_3 - B_4)(p, m_{\psi_i}, m_{\psi_j})$$
$$+ (g_L^{\bar{\psi}_j\psi_iV'}g_R^{\bar{\psi}_i\psi_jV^*} + g_R^{\bar{\psi}_j\psi_iV'}g_L^{\bar{\psi}_i\psi_jV^*})m_{\psi_i}m_{\psi_j}B_0(p, m_{\psi_i}, m_{\psi_j})\Bigg\}, \quad \text{(A.10)}$$

where $g_{L,R}^{\bar{\psi}_i\psi_jV}$ is the coupling strength of the vector boson with left-handed or righ-handed fermions. The functions B_3 and B_4 are related with the standard two-point functions by [35]

$$B_3(p, m_1, m_2) = -B_1(p, m_1, m_2) - B_{21}(p, m_1, m_2),$$
$$B_4(p, m_1, m_2) = -m_1^2 B_1(p, m_2, m_1) - m_2^2 B_1(p, m_1, m_2).$$

For the charginos and neutralinos, the coefficients of their interactions with vector bosons take following forms:

$$g_L^{\bar{\chi}_i^0\tilde{\chi}_j^+W^-} = g\left(-\frac{1}{\sqrt{2}}N_{i3}V_{j2}^* + N_{i2}V_{j1}^*\right), \qquad g_R^{\bar{\chi}_i^0\tilde{\chi}_j^+W^-} = g\left(\frac{1}{\sqrt{2}}N_{i4}^*U_{j2} + N_{i2}^*U_{j1}\right),$$

$$g_L^{\bar{\chi}_i^0\tilde{\chi}_j^0Z} = \frac{g}{2\cos\theta_W}(-N_{i4}N_{j4}^* + N_{i3}N_{j3}^*), \qquad g_R^{\bar{\chi}_i^0\tilde{\chi}_j^0Z} = \frac{g}{2\cos\theta_W}(N_{i4}^*N_{j4} - N_{i3}^*N_{j3}),$$

$$g_L^{\bar{\chi}_i^+\tilde{\chi}_j^+Z} = \frac{g}{\cos\theta_W}\left(-V_{i1}V_{j1}^* - \frac{1}{2}V_{i2}V_{j2}^* + \delta_{ij}\sin^2\theta_W\right), \qquad g_L^{\bar{\chi}_i^+\tilde{\chi}_j^+\gamma} = -e\delta_{ij},$$

$$g_R^{\bar{\chi}_i^+\tilde{\chi}_j^+Z} = \frac{g}{\cos\theta_W}\left(-U_{i1}^*U_{j1} - \frac{1}{2}U_{i2}^*U_{j2} + \delta_{ij}\sin^2\theta_W\right), \qquad g_R^{\bar{\chi}_i^+\tilde{\chi}_j^+\gamma} = -e\delta_{ij}.$$

But as for the contribution from neutralino sector, one should note that, due to the Majorana nature of neutralinos, an addition factor $\frac{1}{2}$ should be multiplied when using above formulae to get neutralino contribution to Z-boson self-energy.

B. Vertex corrections to $Z \to f\bar{f}$ in NMSSM

In this section we present the expressions of the radiative correction to $Z\bar{f}f$ vertex in the NMSSM, namely δv_f and δa_f defined in eq. (2.3). In our calculation we neglect terms proportional to fermion mass except for $f = b$ (bottom quark) where we keep terms proportional to bottom quark Yukawa coupling, $Y_b \sim \frac{m_b}{\cos\beta}$, since those terms may be enhanced by large $\tan\beta$. Throughout this section all Z-boson coupling coefficients, such as δv_f and δa_f, are defined so that the common factor $e/(2\sin\theta_W\cos\theta_W)$ has been extracted.

To neatly present δv_f and δa_f, it is convenient to introduce the quantities δg_λ^f with $\lambda = L, R$, which denote the vertex correction to $Z\bar{f}_\lambda f_\lambda$ interaction and are related with δv_f and δa_f by $\delta v_f = (\delta g_L^f + \delta g_R^f)/2$ and $\delta a_f = (\delta g_L^f - \delta g_R^f)/2$, respectively. δg_λ^f is given by [14]

$$\delta g_\lambda^f = \Gamma_{f_\lambda}(m_Z^2) - g_\lambda^{Z\bar{f}f}\Sigma_{f_\lambda}(m_f^2) - 2\delta_{\lambda L}a_f\frac{\cos\theta_W}{\sin\theta_W}\frac{\Sigma^{\gamma Z}(0)}{m_Z^2}, \quad \text{(B.1)}$$

where Γ_{f_λ} is the unrenormalized vertex correction to $Z\bar{f}_\lambda f_\lambda$ interaction, the second term on the r.h.s. denotes the counter term arising from the fermion f_λ self-energy, and the last term is the counter term from the vector boson self-energy.

Assuming the interaction between scalars ϕ_i with Z boson takes the form $\Gamma^{\phi_i^* \phi_j Z} = g^{\phi_i^* \phi_j Z}(p_{\phi_i} + p_{\phi_j})$, we can write down $\Sigma_{f_\lambda}(m_f^2)$ and the vertex function $\Gamma_{f_\lambda}(q^2)$ mediated by a fermion ψ and a scalar ϕ in a compact generic notation as

$$(4\pi)^2 \Sigma_{f_\lambda}(p_f^2) = C_g \left| g_\lambda^{\bar{\psi}_j f \phi_i^*} \right|^2 \left(B_0 + B_1 \right)(p_f, m_{\phi_i}, m_{\psi_j}), \tag{B.2}$$

$$(4\pi)^2 \Gamma_{f_\lambda}(q^2) = -C_g \Bigg\{ \left(g_\lambda^{\bar{\psi}_j f \phi_k^*} \right)^* g_\lambda^{\bar{\psi}_i f \phi_k^*} \bigg[g_\lambda^{\bar{\psi}_j \psi_i Z} m_{\psi_i} m_{\psi_j} C_0$$

$$+ g_{-\lambda}^{\bar{\psi}_j \psi_i Z} \left\{ -q^2(C_{12} + C_{23}) - 2C_{24} + \frac{1}{2} \right\} \bigg] (p_{\bar{f}}, p_f, m_{\psi_i}, m_{\phi_k}, m_{\psi_j})$$

$$- \left(g_\lambda^{\bar{\psi}_k f \phi_i^*} \right)^* g_\lambda^{\bar{\psi}_k f \phi_j^*} g^{\phi_i^* \phi_j Z} 2C_{24}(p_{\bar{f}}, p_f, m_{\phi_j}, m_{\psi_k}, m_{\phi_i}) \Bigg\}. \tag{B.3}$$

Here C_g is $4/3$ for the gluino contribution ($\psi = gluino$) and 1 for the others. The chirality index $-\lambda$ follows the rule: $-L = R, -R = L$.

If f is a lepton, the following combination of $\{\psi, \phi\}$ contribute to the vertex:

- Chargino correction:

$$\{\psi, \phi\} = \{\tilde{\chi}^-, \tilde{\nu}\}:$$
$$g_L^{\bar{\tilde{\chi}}_j^- l \tilde{\nu}^*} = -g V_{j1}^*; \qquad g_R^{\bar{\tilde{\chi}}_j^- l \tilde{\nu}^*} = 0; \qquad g^{\tilde{\nu}^* \tilde{\nu} Z} = -1;$$
$$g_L^{\bar{\tilde{\chi}}_j^- \tilde{\chi}_i^- Z} = 2\left(U_{i1}^* U_{j1} + \frac{1}{2} U_{i2}^* U_{j2} - \delta_{ij} \sin^2\theta_W \right);$$
$$g_R^{\bar{\tilde{\chi}}_j^- \tilde{\chi}_i^- Z} = 2\left(V_{i1} V_{j1}^* + \frac{1}{2} V_{i2} V_{j2}^* - \delta_{ij} \sin^2\theta_W \right); \tag{B.4}$$

- Neutralino correction:

$$\{\psi, \phi\} = \{\tilde{\chi}^0, \tilde{l}\}:$$
$$g_L^{\bar{\tilde{\chi}}_j^0 l \tilde{l}_\alpha^*} = \frac{g}{\sqrt{2}} R_{\alpha 1}^{\tilde{l}}(N_{j2}^* + \tan\theta_W N_{j1}^*); \qquad g_R^{\bar{\tilde{\chi}}_j^0 l \tilde{l}_\alpha^*} = -\sqrt{2} g R_{\alpha 2}^{\tilde{l}} \tan\theta_W N_{j1};$$
$$g_L^{\bar{\tilde{\chi}}_j^0 \tilde{\chi}_i^0 Z} = -N_{j4} N_{i4}^* + N_{j3} N_{i3}^*; \qquad g_R^{\bar{\tilde{\chi}}_j^0 \tilde{\chi}_i^0 Z} = N_{j4}^* N_{i4} - N_{j3}^* N_{i3};$$
$$g^{\tilde{l}_\alpha^* \tilde{l}_\beta Z} = (1 - 2\sin^2\theta_W) R_{\alpha 1}^{\tilde{l}} R_{\beta 1}^{\tilde{l}*} - 2\sin^2\theta_W R_{\alpha 2}^{\tilde{l}} R_{\beta 2}^{\tilde{l}*}; \tag{B.5}$$

If f is the bottom quark, the following combination of $\{\psi, \phi\}$ contribute to the vertex:

- Chargino correction:

$$\{\psi, \phi\} = \{\tilde{\chi}^-, \tilde{t}\}:$$
$$g_L^{\bar{\tilde{\chi}}_j^- b \tilde{t}_\alpha^*} = g(-R_{\alpha 1}^{\tilde{t}} V_{j1}^* + Y_t R_{\alpha 2}^{\tilde{t}} V_{j2}^*); \quad g_R^{\bar{\tilde{\chi}}_j^- b \tilde{t}_\alpha^*} = g R_{\alpha 1}^{\tilde{t}} Y_b U_{j2};$$

$$g^{\tilde{t}^*_\alpha \tilde{t}_\beta Z} = \left(-1 + \frac{4}{3} \sin^2 \theta_W \right) R^{\tilde{t}}_{\alpha 1} R^{\tilde{t}^*}_{\beta 1} + \frac{4}{3} \sin^2 \theta_W R^{\tilde{t}}_{\alpha 2} R^{\tilde{t}^*}_{\beta 2}; \qquad (\text{B.6})$$

Note that in order to write the couplings in a neat form, we define $Y_t = m_t/\sqrt{2}m_W \sin\beta$, and $Y_b = m_b/\sqrt{2}m_W \cos\beta$. Such definitions differ from their conventional definitions by a factor g. We adopt such a convention throughout our paper.

- Neutralino correction:

$$\{\psi, \phi\} = \{\tilde{\chi}^0, \tilde{b}\} :$$
$$g^{\bar{\tilde{\chi}}^0_j b \tilde{b}^*_\alpha}_L = g\left(\frac{\sqrt{2}}{2} R^{\tilde{b}}_{\alpha 1} \left(N^*_{j2} - \frac{1}{3} \tan\theta_W N^*_{j1} \right) - Y_b R^{\tilde{b}}_{\alpha 2} N^*_{j4} \right);$$
$$g^{\bar{\tilde{\chi}}^0_j b \tilde{b}^*_\alpha}_R = -g\left(R^{\tilde{b}}_{\alpha 1} Y_b N_{j4} + \frac{\sqrt{2}}{3} R^{\tilde{b}}_{\alpha 2} \tan\theta_W N_{j1} \right);$$
$$g^{\tilde{b}^*_\alpha \tilde{b}_\beta Z} = \left(1 - \frac{2}{3} \sin^2\theta_W \right) R^{\tilde{b}}_{\alpha 1} R^{\tilde{b}^*}_{\beta 1} - \frac{2}{3} \sin^2\theta_W R^{\tilde{b}}_{\alpha 2} R^{\tilde{b}^*}_{\beta 2}; \qquad (\text{B.7})$$

- Gluino correction:

$$\{\psi, \phi\} = \{\tilde{g}, \tilde{b}\} :$$
$$g^{\bar{\tilde{g}}b\tilde{b}^*_\alpha}_L = -\sqrt{2}g_s R^{\tilde{b}}_{\alpha 1}; \quad g^{\bar{\tilde{g}}b\tilde{b}^*_\alpha}_R = \sqrt{2}g_s R^{\tilde{b}}_{\alpha 2}; \qquad (\text{B.8})$$

- Charged Higgs contribution:

$$\{\psi, \phi\} = \{t, H^-\} :$$
$$g^{\bar{t}b(H^-)^*}_L = \frac{gm_t}{\sqrt{2}m_W} \cot\beta; \qquad\qquad g^{\bar{t}b(H^-)^*}_R = \frac{gm_b}{\sqrt{2}m_W} \tan\beta;$$
$$g^{\bar{t}tZ}_L = -\left(1 - \frac{4}{3} \sin^2\theta_W \right); \qquad\qquad g^{\bar{t}tZ}_R = \frac{4}{3} \sin^2\theta_W;$$
$$g^{(H^-)^* H^- Z} = \cos 2\theta_W \qquad\qquad\qquad\qquad\qquad (\text{B.9})$$

- Neutral Higgs contribution:

$$\{\psi, \phi\} = \{b, (h, a, G^0)\} :$$
$$g^{\bar{b}bh_i}_L = -\frac{gm_b}{2m_W \cos\beta} S_{i2}; \qquad\qquad\qquad g^{\bar{b}bh_i}_R = -\frac{gm_b}{2m_W \cos\beta} S_{i2};$$
$$g^{\bar{b}ba_i}_L = -\frac{igm_b}{2m_W \cos\beta} P_{i2} = -\frac{igm_b}{2m_W} P'_{i1} \tan\beta;$$
$$g^{\bar{b}ba_i}_R = \frac{igm_b}{2m_W \cos\beta} P_{i2} = \frac{igm_b}{2m_W} P'_{i1} \tan\beta;$$
$$g^{\bar{b}bG^0}_L = -\frac{igm_b}{2m_W}; \qquad\qquad\qquad\qquad g^{\bar{b}bG^0}_R = \frac{igm_b}{2m_W};$$
$$g^{\bar{b}bZ}_L = \left(1 - \frac{2}{3} \sin^2\theta_W \right); \qquad\qquad\qquad g^{\bar{b}bZ}_R = -\frac{2}{3} \sin^2\theta_W;$$
$$g^{h^*_i a_j Z} = -i(S_{i2} P_{j2} - S_{i1} P_{j1}) = -i(S_{i2} \sin\beta - S_{i1} \cos\beta) P'_{j1},$$

$$g^{a_j^* h_i Z} = i(S_{i2} P_{j2} - S_{i1} P_{j1}) = i(S_{i2} \sin\beta - S_{i1} \cos\beta) P'_{j1},$$

$$g^{h_i^* G^0 Z} = -i(S_{i2} \cos\beta + S_{i1} \sin\beta),$$

$$g^{G^{0*} h_i Z} = i(S_{i2} \cos\beta + S_{i1} \sin\beta). \tag{B.10}$$

Note that in the above formulas we did not include the contribution to δg_λ from the loop of $\{t, G^-\}$. Such contribution alone is UV-convergent and should be attributed to the SM radiative effects. This situation is quite different for the neutral Higgs contribution where the effects of the loops of $\{b, G^0\}$ are UV divergence and must be included with other neutral Higgs contribution to get an finite result.

If f is the charm quark, the following combination of $\{\psi, \phi\}$ contribute to the vertex:

- Chargino correction:

$$\{\psi, \phi\} = \{\tilde\chi^+, \tilde s\} :$$

$$g_L^{\bar{\tilde\chi}_j^+ c \tilde s_\alpha^*} = -g R_{\alpha 1}^{\tilde s} U_{j1}^*; \qquad g_R^{\bar{\tilde\chi}_j^+ c \tilde s_\alpha^*} = 0;$$

$$g_L^{\bar{\tilde\chi}_j^+ \tilde\chi_i^+ Z} = -2\left(V_{i1}^* V_{j1} + \frac{1}{2} V_{i2}^* V_{j2} - \delta_{ij} \sin^2\theta_W\right);$$

$$g_R^{\bar{\tilde\chi}_j^+ \tilde\chi_i^+ Z} = -2\left(U_{i1} U_{j1}^* + \frac{1}{2} U_{i2} U_{j2}^* - \delta_{ij} \sin^2\theta_W\right);$$

$$g^{\tilde s_\alpha^* \tilde s_\beta Z} = \left(1 - \frac{2}{3}\sin^2\theta_W\right) R_{\alpha 1}^{\tilde s} R_{\beta 1}^{\tilde s *} - \frac{2}{3}\sin^2\theta_W R_{\alpha 2}^{\tilde s} R_{\beta 2}^{\tilde s *}; \tag{B.11}$$

- Neutralino correction:

$$\{\psi, \phi\} = \{\tilde\chi^0, \tilde c\} :$$

$$g_L^{\bar{\tilde\chi}_j^0 c \tilde c_\alpha^*} = -\frac{g}{\sqrt{2}} R_{\alpha 1}^{\tilde c}\left(N_{j2}^* + \frac{1}{3}\tan\theta_W N_{j1}^*\right);$$

$$g_R^{\bar{\tilde\chi}_j^0 c \tilde c_\alpha^*} = \frac{2\sqrt{2}g}{3} R_{\alpha 2}^{\tilde c}\tan\theta_W N_{j1};$$

$$g^{\tilde c_\alpha^* \tilde c_\beta Z} = \left(-1 + \frac{4}{3}\sin^2\theta_W\right) R_{\alpha 1}^{\tilde c} R_{\beta 1}^{\tilde c *} + \frac{4}{3}\sin^2\theta_W R_{\alpha 2}^{\tilde c} R_{\beta 2}^{\tilde c *}; \tag{B.12}$$

- Gluino correction:

$$\{\psi, \phi\} = \{\tilde g, \tilde c\} :$$

$$g_L^{\bar{\tilde g} c \tilde c_\alpha^*} = -\sqrt{2} g_s R_{\alpha 1}^{\tilde c}; \qquad g_R^{\bar{\tilde g} c \tilde c_\alpha^*} = \sqrt{2} g_s R_{\alpha 2}^{\tilde c}; \tag{B.13}$$

The above expressions then suffice to calculate all the $Z f_\alpha \bar{f}_\alpha$ vertex corrections δg_α^f. Summation should be taken over all non-vanishing coupling combinations, such as over the indices of sfermions, charginos, neutralinos, scalar Higgs and pseudo-scalar Higgs.

C. NMSSM contributions to the μ-decay

In the NMSSM the flavor-dependent correction to the decay $\mu \to \nu_\mu e \bar{\nu}_e$ mainly comes from the loops mediated by gauginos, and the corrected amplitude can be written as [26]

$$M = M_B \left(1 + 2\delta^{(v)} + \delta^{(b)} \right), \qquad (C.1)$$

where M_B is the Born amplitude, $\delta^{(v)}$ is the vertex correction for either $\bar{e}\nu_e W$ interaction or $\bar{\mu}\nu_\mu W$ interaction (since we assume the mass degeneracy for the first two generations of sleptons, the two corrections are same), and $\delta^{(b)}$ denotes box diagram correction.

(1) Vertex corrections

Similar to eq. (B.1), the correction to $\bar{f}_1 f_2 W$ interaction can be expressed as

$$g_L^{\bar{f}_1 f_2 W} \delta^{(v)} = \Gamma^{\bar{f}_1 f_2 W}(q^2) - \frac{1}{2} g_L^{\bar{f}_1 f_2 W} \left\{ \Sigma_{f_1}(m_{f_1}^2) + \Sigma_{f_2}(m_{f_2}^2) \right\}. \qquad (C.2)$$

For the $\bar{e}\nu_e W$ interaction, we have $g_L^{\bar{e}\nu_e W^-} = -\frac{g}{\sqrt{2}}$,

$$
\begin{aligned}
(4\pi)^2 \Sigma_{e_L}(m_e^2) &= |g_L^{\bar{\tilde{\chi}}_i^0 e \tilde{e}_L^*}|^2 (B_0 + B_1)(m_e^2, m_{\tilde{e}_L}, m_{\tilde{\chi}_i^0}) \\
&\quad + |g_L^{\bar{\tilde{\chi}}_j^- e \tilde{\nu}_e^*}|^2 (B_0 + B_1)(m_e^2, m_{\tilde{\nu}_e}, m_{\tilde{\chi}_j^-}), \\
(4\pi)^2 \Sigma_{\nu_e}(m_{\nu_e}^2) &= |g_L^{\bar{\tilde{\chi}}_i^0 \nu_e \tilde{\nu}_e^*}|^2 (B_0 + B_1)(m_{\nu_e}^2, m_{\tilde{\nu}_e}, m_{\tilde{\chi}_i^0}) \\
&\quad + |g_L^{\bar{\tilde{\chi}}_j^+ \nu_e \tilde{e}_L^*}|^2 (B_0 + B_1)(m_{\nu_e}^2, m_{\tilde{e}_L}, m_{\tilde{\chi}_j^+}), \\
(4\pi)^2 \Gamma_{\bar{e}\nu_e W^-} &= -(g_L^{\bar{\tilde{\chi}}_i^0 e \tilde{e}_L^*})^* g_L^{\bar{\tilde{\chi}}_j^+ \nu_e \tilde{e}_L^*} \\
&\quad \times \left\{ g_L^{\bar{\tilde{\chi}}_i^0 \tilde{\chi}_j^+ W} m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^+} C_0 + g_R^{\bar{\tilde{\chi}}_i^0 \tilde{\chi}_j^+ W} \left(-2C_{24} + \frac{1}{2} \right) \right\} \\
&\quad \times (p_{\nu_e}, p_e, m_{\tilde{\chi}_j^+}, m_{\tilde{e}_L}, m_{\tilde{\chi}_i^0}) \\
&\quad - (g_L^{\bar{\tilde{\chi}}_j^- e \tilde{\nu}_e^*})^* g_L^{\bar{\tilde{\chi}}_i^0 \nu_e \tilde{\nu}_e^*} \\
&\quad \times \left\{ g_L^{\bar{\tilde{\chi}}_j^- \tilde{\chi}_i^0 W} m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^-} C_0 + g_R^{\bar{\tilde{\chi}}_j^- \tilde{\chi}_i^0 W} \left(-2C_{24} + \frac{1}{2} \right) \right\} \\
&\quad \times (p_{\nu_e}, p_e, m_{\tilde{\chi}_i^0}, m_{\tilde{\nu}_e}, m_{\tilde{\chi}_j^-}) \\
&\quad + 2 (g_L^{\tilde{\chi}_i^0 e \tilde{e}_L^*})^* g_L^{\tilde{\chi}_i^0 \nu_e \tilde{\nu}_e^*} g^{\tilde{e}_L^* \tilde{\nu}_e W} C_{24}(p_{\nu_e}, p_e, m_{\tilde{\nu}_e}, m_{\tilde{\chi}_i^0}, m_{\tilde{e}_L}). \quad (C.3)
\end{aligned}
$$

In the above equations, summation over $i = 1$ to 5 ($\tilde{\chi}_i^0$) and $j = 1$ to 2 ($\tilde{\chi}_j^\pm$) is implied. The coupling g_L takes the following forms

$$g_L^{\bar{\tilde{\chi}}_i^0 \nu_e \tilde{\nu}_e^*} = \frac{g}{\sqrt{2}}(N_{i1}^* \tan\theta_W - N_{i2}^*); \qquad g_L^{\bar{\tilde{\chi}}_i^0 e \tilde{e}_L^*} = \frac{g}{\sqrt{2}}(N_{i1}^* \tan\theta_W + N_{i2}^*);$$

$$g_L^{\bar{\tilde{\chi}}_j^+ \nu_e \tilde{e}_L^*} = -g U_{j1}^*; \qquad\qquad\qquad g_L^{\bar{\tilde{\chi}}_j^- e \tilde{\nu}_e^*} = -g V_{j1}^*$$

$$g_L^{\bar{\tilde{\chi}}_i^0 \tilde{\chi}_j^+ W} = \frac{g}{\sqrt{2}}(\sqrt{2}V_{j1}^* N_{i2} - V_{j2}^* N_{i3}); \quad g_R^{\bar{\tilde{\chi}}_i^0 \tilde{\chi}_j^+ W} = \frac{g}{\sqrt{2}}(\sqrt{2}U_{j1} N_{i2}^* + U_{j2} N_{i4}^*);$$

$$g_L^{\bar{\tilde{\chi}}_j^- \tilde{\chi}_i^0 W} = -g_R^{\bar{\tilde{\chi}}_i^0 \tilde{\chi}_j^+ W}; \qquad\qquad g_R^{\bar{\tilde{\chi}}_j^- \tilde{\chi}_i^0 W} = -g_L^{\bar{\tilde{\chi}}_i^0 \tilde{\chi}_j^+ W}; \qquad g^{\tilde{e}_L^* \tilde{\nu}_e W} = -\frac{g}{\sqrt{2}},$$

and for the three-point loop functions, since we take their external momentum to be zero, their expressions are greatly simplified:

$$C_0(m_1, m_2, m_3) = -\frac{1}{m_3^2}\left\{ -\frac{(1+a)\ln(1+a)}{ab} + \frac{(1+a+b)\ln(1+a+b)}{(a+b)b} \right\}$$

$$C_{24}(m_1, m_2, m_3) = \frac{\Delta}{4} - \frac{1}{4}\ln\frac{m_3^2}{\mu^2} - \frac{1}{2}\left\{ \frac{-2(1+a)^2\ln(1+a)}{4ab} \right.$$
$$\left. + \frac{-3b(a+b) + 2(1+a+b)^2\ln(1+a+b)}{4b(a+b)} \right\}$$

with $a = \frac{m_2^2 - m_3^2}{m_3^2}$ and $b = \frac{m_1^2 - m_3^2}{m_3^2}$.

(2) Box corrections

The box diagram contributions to the $\mu \to \nu_\mu e \bar{\nu}_e$ amplitude can be expressed as

$$iT = i\left\{ M(1) + M(2) + M(3) + M(4) \right\} \bar{u}_e \gamma^\mu P_L v_{\nu_e} \bar{u}_{\nu_\mu} \gamma^\mu P_L u_\mu. \qquad (C.4)$$

Taking into account the normalization of the tree-level amplitude, $-g^2/2M_W^2$, the box diagram contributions can be written as

$$\delta^{(b)} = -\frac{2M_W^2}{g^2} \sum_{i=1}^{4} M(i). \qquad (C.5)$$

with each $M(i)$ given by

$$16\pi^2 M(1) = (g_L^{\bar{\tilde{\chi}}_i^0 e \tilde{e}_L^*})^* g_L^{\bar{\tilde{\chi}}_i^0 \mu \tilde{\mu}_L^*} (g_L^{\bar{\tilde{\chi}}_j^+ \nu_\mu \tilde{\mu}_L^*})^* g_L^{\bar{\tilde{\chi}}_j^+ \nu_e \tilde{e}_L^*} D_{27}(m_{\tilde{\mu}_L}, m_{\tilde{e}_L}, m_{\tilde{\chi}_j^+}, m_{\tilde{\chi}_i^0}),$$

$$16\pi^2 M(2) = (g_L^{\bar{\tilde{\chi}}_j^- e \tilde{\nu}_e^*})^* g_L^{\bar{\tilde{\chi}}_j^- \mu \tilde{\nu}_\mu^*} (g_L^{\bar{\tilde{\chi}}_i^0 \nu_\mu \tilde{\nu}_\mu^*})^* g_L^{\bar{\tilde{\chi}}_i^0 \nu_e \tilde{\nu}_e^*} D_{27}(m_{\tilde{\nu}_\mu}, m_{\tilde{\nu}_e}, m_{\tilde{\chi}_j^-}, m_{\tilde{\chi}_i^0}),$$

$$16\pi^2 M(3) = \frac{1}{2} m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^-} g_L^{\bar{\tilde{\chi}}_j^+ \nu_e \tilde{e}_L^*} g_L^{\bar{\tilde{\chi}}_j^- \mu \tilde{\nu}_\mu^*} (g_L^{\bar{\tilde{\chi}}_i^0 \nu_\mu \tilde{\nu}_\mu^*})^* (g_L^{\bar{\tilde{\chi}}_i^0 e \tilde{e}_L^*})^* D_0(m_{\tilde{\nu}_\mu}, m_{\tilde{e}_L}, m_{\tilde{\chi}_j^-}, m_{\tilde{\chi}_i^0}),$$

$$16\pi^2 M(4) = \frac{1}{2} m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^-} g_L^{\bar{\tilde{\chi}}_i^0 \nu_e \tilde{\nu}_e^*} g_L^{\bar{\tilde{\chi}}_i^0 \mu \tilde{\mu}_L^*} (g_L^{\bar{\tilde{\chi}}_j^+ \nu_\mu \tilde{\mu}_L^*})^* (g_L^{\bar{\tilde{\chi}}_j^- e \tilde{\nu}_e^*})^* D_0(m_{\tilde{\mu}_L}, m_{\tilde{\nu}_e}, m_{\tilde{\chi}_j^-}, m_{\tilde{\chi}_i^0}).$$

Here all the D-functions are evaluated at the zero momentum-transfer limit. Noting the fact that $m_{\tilde{\mu}_L} \simeq m_{\tilde{e}_L} \simeq m_{\tilde{\nu}_\mu} \simeq m_{\tilde{\nu}_e}$, we may write the D functions as

$$D_0(m_1, m_1, m_2, m_3) = \frac{1}{m_3^4}\left\{ \frac{-(1+a)\ln(1+a)}{ab^2} \right.$$
$$\left. + \frac{-b(a+b) + ((a+b)(1+a+b) + b)\ln(1+a+b)}{b^2(a+b)^2} \right\},$$

$$D_{27}(m_1, m_1, m_2, m_3) = -\frac{1}{2m_3^2}\left\{ \frac{(1+a)^2\ln(1+a)}{2ab^2} \right.$$
$$\left. - \frac{(1+a+b)(-b(a+b) + ((a+b)(1+a)+b)\ln(1+a+b))}{2b^2(a+b)^2} \right\}.$$

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